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Ordinal benefits vs economic benefits as a reference guide for policy decision making. The case of hydrogen technologies

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ABSTRACT

The paper presents an Investment Evaluation Method in Energetic–Economic–Environmental field which is particularly indicated for Hydrogen Technologies because it enables us to account not only for the traditional economic return and the possible negative externalities (damages), but also for: i) the *induced economic benefits* at a social level; ii) those *positive* (and negative) *externalities* usually considered as being estimable in economic terms as "proxies"; iii) and, finally, *Ordinal Benefits*. That is those Benefits which are never ever reducible to a simple monetary value, nonetheless they can always be object of a possible estimation, still in economic terms, by means of values understood as a "cipher".

The advantages of a Decision Making Process based on Ordinal Benefits (vs traditional economic benefits) will be shown with reference to the evaluation of well-calibrated Incentives concerning Hydrogen Fuel Cells, both under *static* and *dynamic* conditions. Such an evaluation criterion, which is preferentially based on the estimated external Benefits to be "remunerated" rather than on possible *damages* to be *internalized*, represents a valid reference guide for a Policy Maker. This precisely because it is always orientated toward the genesis of the *Maximum Ordinality Excess.*

1. Introduction

The investment evaluation in Energetic–Economic–Environmental field is generally performed on the basis of a widely-diffused criterion according to which, in principle, every activity should be self-remunerative. Very rarely the associated (positive and negative) externalities are accounted for. The former because, for their intrinsic nature, are difficult to be remunerated for. The latter because, on the basis of the above-mentioned criterion, strictly speaking are not part of the firm economic balance.

Furthermore, even when state incentives are foreseen (for a specific class of productive activities), their comprehensive amount is always decided on the basis of a strictly economic criterion, that is, their potential return on gross domestic product (GDP).

In such a context, generally much wider and variegated than here simply delineated, it could be worth considering the availability of an evaluation method able to account not only for the traditional economic return (in a firm perspective), not even for the sole negative externalities (damages), but also, and in particular, for: i) the *induced economic benefits* at a social level; ii) those *positive externalities* (in addition to negative ones) which are usually considered as being estimable in economic terms as "proxies"; iii) and last, but not least, *Ordinal Benefits*. That is those Benefits which are never ever reducible to a simple monetary value, nonetheless they can always be object of a possible estimation, still in economic terms, by means of values understood as a "cipher".

2. Fundamental bases of the adopted methodology

The methodology here considered, both in steady state and dynamic conditions, is always based on the *Maximum Em-Power Principle*, proposed by H. T. Odum [1,2]. This Principle, in fact, asserts that: "every system reaches its optimum working conditions when it maximizes the *total* processed Emergy" (including that of its *surrounding habitat*) [3]. This is why the method adopted, specifically finalized to the strategic evaluation of a given productive sector (typically a firm), takes into consideration the Benefits that originate from the *inter-exchanges* from the considered sector and the *surrounding* sectors (understood as *habitat*). This is also the basic reason for its name: *Four-Sector Diagram Of Benefits* (FSDOB). In fact, on the basis of such a perspective, the method guides to the evaluation of Benefits pertaining to the main four distinct "Subjects" (or Sectors) which are usually involved in any productive activity: Benefits to the *Firm* (deriving from the Production process), Benefits to *Society* (deriving from the new Product

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generated), Benefits to the *Environment as a Source*, Benefits to the *Environment as a Sink*.

However, differently from the traditional economic evaluations, the Benefits inter-exchanged (in steady state conditions) are evaluated in Emergy terms (see Eq. (1)), in order to highlight those strategic decisions which are mostly adherent to the M. Em-Power Principle.

The original concept of Emergy, in fact, introduces a profound novelty in Thermodynamics, that is: *there are processes which cannot be considered as being pure "mechanisms"* (see also Appendix). This is equivalent to say that they are not describable in mere functional terms, because their outputs show an unexpected "excess" (with respect to their pertinent inputs), which can be termed as Quality (with a capital Q) exactly because it is no longer understood as a simple "property" or a "characteristic" of a given phenomenon, but it is recognized as being any *emerging* "property" (from the considered process) *never ever reducible* to its phenomenological premises or to our traditional mental categories.

The *Maximum Em-Power Principle* then suggests we focus our attention on those processes which can be considered as more specifically generative. Among them (as the same Odum points out) there are three fundamental processes (*co-production*, *inter-action*, *feed-back*) in which such an aspect is particularly evident. These processes, in fact, when analyzed under steady state conditions, can more appropriately be described by means of a particular *non-conservative* Algebra [4]. This leads to the introduction of the concept of *Transformity*, which allows us to define Emergy as the product of a given quantity of available Energy (represented by Exergy), by the product of its corresponding Quality (expressed by Transformity).

We can thus write:

$$Emergy = Transformity(\text{Energy Quality})\ by\ Exergy\ (\text{Energy quantity}). \tag{1}$$

Transformity (Tr), in turn, can also be articulated in two scalar factors

$$Tr = Tr_{\phi} \cdot Tr_{ex} \tag{2}$$

where Tr_{ex} (dissipative Transformity) accounts for the losses of Exergy used up during the generation process of a given product or service, whereas Tr_{ϕ} (generative Transformity) accounts for the ever-increasing content of *Ordinal Information* due to those generative Processes [5]. This is why Transformity, although represented as an algebraic cardinal "factor", is always understood in an *Ordinal sense* (the concepts of *Ordinal* and *Ordinality* will be given in the next paragraph).

3. The concept of ordinality

The concept of Ordinality takes origin from the research for a general mathematical formulation of the Maximum Em-Power Principle under dynamic conditions. This correspondently required the consequential generalization of the Rules of Emergy Algebra and, contextually, the introduction of a *new concept of derivative*, the "incipient" derivative, apt to formally account for the *a-functional* characteristics of the considered processes. This new derivative was termed as "incipient" (or *prior* derivative) because it focuses on the various processes in their act of being born. Its mathematical definition (given in Appendix) enables us to represent the three basic processes (*co-production*, *inter-action*, *feed-back*) in terms of *fractional derivatives* of order 1/2, 2, and {2/2} respectively (see Appendix). In such a case the order of derivation is termed as Ordinality, because the resulting "functions" ("binary", "duet", and "binary–duet" functions, respectively) are structured in such a way as to show an "excess" of Information, which is never reducible to its phenomenological premises or to our traditional mental categories [5,6].

One of the major consequences of such a new mathematical description is that an "incipient" differential equation (or even a system of differential equations), which describes any given process, will always have a solution characterized by both cardinality and Ordinality. The solution in fact can always be structured as

$$[f(t)]^{l,(\frac{m}{n})} \tag{3}$$

where: $[f(t)]^l$ represents its *cardinality*, whereas its *Ordinality* (m/n) corresponds to the order of the basic fractional derivative $(1/n)$ multiplied by the maximum non-linearity degree (m) of the considered generating equation.

4. Ordinal externalities

The concept of Ordinality is not strictly limited to physical–biological processes analyzed in Thermodynamics, but it can easily be recognized in several other disciplines. In particular, in Economics [7].

Let us start from the concept of transaction analyzed by Odum [1,2] (see Fig. 1a).

In such a case money and goods (exchanged in counter current), when analyzed *in Emergy terms*, do not reduce their meaning to mere physical–economic concepts. In fact the Emergy associated to any product/service (i) can be written as

$$Em_i = Tr_{\phi,i} \cdot Tr_{ex,i} \cdot Ex_i \tag{4}$$

where $Tr_{\phi,i}$ (generative Transformity) is understood as a "cipher" of the *Ordinality* "vehicled" by a given product/service, whereas $Tr_{ex,i}$ (as usual) accounts for the losses of Exergy (Ex_i) used up during the generation process of the same product/service (see also Eq. (2)).

Consequently any Transaction (see Fig. 1b) represents an exchange of different Emergies, both in terms of *cardinality* and *Ordinality*. However, as a consequence of an ever-present *disequilibrium* between the exchanged Emergies (and related *Ordinalities*), only when the two Subjects operate in consonance with the Maximum Em-Power Principle does the transaction: i) become a true *transactive* inter-action; ii) present a reciprocal *increase* in Ordinality (as a consequence of an actual *inter-action*). In this sense Ordinal Externality can be defined as "the excess of Ordinality emerging from a *real* transaction relationship".

In this paper, however, we do not consider either the generative Transformity ($Tr_{\phi,2}$) associated to money (such as, for instance, that pertaining to state incentives), or the total energy spent to produce it (represented by $Tr_{ex,2}\ Ex_2$). This is because the main aim of the paper is to show the intrinsic limitations of traditional investment criteria. These in fact systematically neglect not only the thermodynamic value ($Tr_{ex,1}\ Ex_1$) of natural resources (usually considered as being simply "given"), but also, and in a special way, all those *Benefits* which are related to the Generative Processes that gave origin to any natural product (or human artifact), and which are thus proportional to the *generative* Transformity $Tr_{\phi,1}$.

On the basis of such concepts we can now present the FSDOB evaluation method. First in steady state conditions (corresponding to its original version) and then under dynamic conditions. The method in fact was first developed in collaboration with the University of Padua and the University of Siena [8,9], and more recently transformed into a pseudo-dynamic computer code [10]. The dynamic version then represents a further improvement of the same.

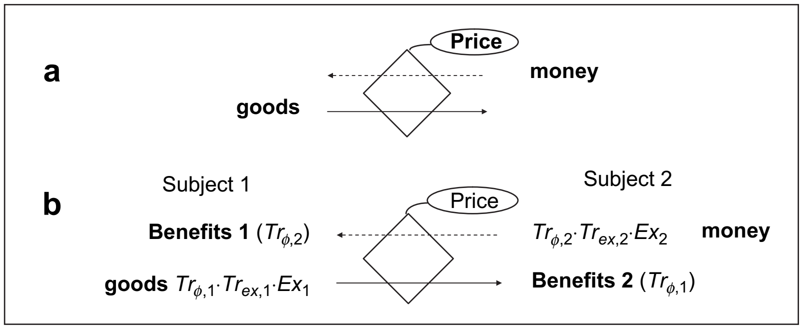

Fig. 1. Ordinal Externality understood as an "excess of Ordinality".

5. The Four-Sector Diagram of Benefits

The method (described in detail in [8]) can be synthesized as follows. Each sector is identified by two axes which point out the fundamental features of its input/output properties respectively. Each coordinate axis is characterized by five Indicators (listed in Table 1), which are normalized on the basis of appropriate reference values, specific to the typology of the process investigated (in brackets, in Table 2). Each normalized Indicator can be appropriately weighted in order to account for its specific influence. The only condition for the specific weights ($w_{ij}^{(k)}$) is that

$$\sum_{j=1}^{5} w_{ij}^{(k)} = 1 \quad \text{for } i = 1,2,\ldots 8 \quad k = 1,2,\ldots m \tag{5}$$

where i = axis, j = sequential order of the Indicator, k = sequential order of the Plant each time considered. The weighted average of each axis ($\overline{w}_i^{(k)}$) is then evaluated (see Table 2) by assuming that all the normalized Indicators have, as a basic reference level, the same weights (namely $w_{ij}^{(k)} = 1/5 = 0.20$). By taking into account that the Decision Maker will probably adopt a differentiated distribution of weights, the method foresees the evaluation of the maximum (positive and negative) variations ($\Delta\overline{w}_i^{(k)}$) with respect to the previous values, in correspondence to a predefined margin of confidence. The latter is defined as the ratio between the maximum and minimum modified weights (always in the respect of condition (5)) and it usually equals 500%. On the basis of the values obtained, a summary diagram can be consequently drawn (such as those in Figs. 2 and 3).

The barycentres of the "circles" represent the average values $\overline{w}_i^{(k)}$. The inner radius ($\overline{r}_g$), defined as

$$\overline{r}_g = \max\left(\left(\frac{1}{8}\sum_{i=1}^{8} \overline{w}_i^{(k)}\right) \cdot \left(\frac{1}{8}\sum_{i=1}^{8} \Delta\overline{w}_i^{(k)}\right)\right), \tag{6}$$

corresponds to a maximum variation evaluated at a *global* level, whereas the outer radius ($\overline{r}_l$)

$$\overline{r}_l = \max\left(\overline{w}_i^{(k)} \cdot \left|\Delta\overline{w}_i^{(k)}\right|\right) \quad i = 1,2,\ldots 8; \tag{7}$$

represents (in the same scale) the maximum variation evaluated at a *local* level.

6. A simple case study

The case study here recalled refers to the evaluation of hydrogen Fuel Cells for stationary production of electricity and heat. It was specifically performed with reference to Agenzia Territoriale per l'Edilizia Residenziale (ATER) – Territorial Agency for Public Building Patrimony (in Rome) – which is responsible for the management of 50,000 flats and 1,800 business premises. More precisely, the case study assumes that 15 fuel cells are installed in several buildings to satisfy energy demand from about 1000 households.

Table 1
List of Indicators (I_{ij}) subdivided by Sectors (i = generic axis of the diagram (i = 1–8); j = sequential order of each Indicator in the axis (i) (j = 1–5; $I_{ij,0}$ = reference value).

Sector 1 Benefits from Production (to the Firm)	I_{11} = Plant cost per unit power (€/kW)	I_{21} = Energy efficiency
	I_{12} = Fuel cost per unit product (€/kWhex)	I_{22} = Exergy efficiency
	I_{13} = Labour cost per unit product (€/kWhex)	I_{23} = Raw Energy conversion coefficient
	I_{14} = Maintenance cost per unit product (€/kWhex)	I_{24} = Transformity of the product (seJ/J)
	I_{15} = Cost of NOx uptake device per unit product (€/kWhex)	I_{25} = Profit Index
Sector 2 Benefits for the Environment as a "Sink"	I_{31} = co-generated heat/total heat supplied	I_{41} = Global Warming (CO_2 release) (kg/MWh)
	I_{32} = Cost of CO_2 sequestration and storage (€/ton)	I_{42} = CO_2 Emission costs at a local level (€/kWh)
	I_{33} = Cost of NOx uptake (€/ton)	I_{43} = CO_2 Emission costs at a global level (€/kWh)
	I_{34} = Reuse of uptaken materials (%)	I_{44} = Cost of NOx emissions (acidification) (€/kWh)
	I_{45} = Fraction of recycle after Decommissioning (%)	I_{45} = Cost of NOx emissions (via ozone) (€/kWh)
Sector 3 Benefits from the Product (to the society)	$I_{51} = \sum_{k=1}{}^{4} ()_k / Inv$ (economic benefit per unit Investment)	$I_{61} = \pi_4/\pi_2$ (benefit to Economy / product cost)
	$I_{52} = EYR^*$ (process economic amplification	$I_{62} = \pi_5/\pi_2$ (feed-back benefits/product cost)
	$I_{53} = Tr_{pd}/Tr_{pc}$ (product benefit per typology of process)	$I_{63} = \pi_6/\pi_2$ (I_{62} at net of local damages)
	$I_{54} = (F' \cdot EYR_f - Inv)/Inv$	$I_{64} = \pi_7/\pi_2$ (I_{63} at net of global damages)
	$I_{55} = \pi_1/\pi_2$ (Firm/citizen financial sustainability)	$I_{65} = \pi_8/\pi_2$ (I_{64} at net of resource consumption)
Sector 4 Benefits for the Environment as a "Source"	I_{71} = ELR (Environmental Loading Ratio)	I_{81} = Emergy Density (seJ/m2)
	I_{72} = EIS (Emergy Index of Sustainability)	I_{82} = Non-renewable Emergy/Total Emergy
	I_{73} = Decrease of biodiversity (%)	I_{83} = Material Intensity, water factor (g/kWh)
	I_{74} = Area supporting the process (m2/MW)	I_{84} = Material Intensity, abiotic factor (g/kWh)
	I_{75} = Actual NOx emission/Law emission limit	I_{85} = Fraction of imported fuel (%)

Table 2
Characteristic parameters of a single fuel cell (500 kW) with state incentives.

Sector	Axis	I_{i1}	I_{i2}	I_{i3}	I_{i4}	I_{i5}	$\overline{w}_i$	
1	1	400 (325)	0.025 (0.02)	1.3E-2 (1.0E-4)	2.0E-4 (2.0E-4)	1.01E-3 (1.0E-3)	0.276	+12% −44%
1	2	79.5 (85.0)	41.0 (75.0)	79.5 (85.0)	1.95E5 (1.0E5)	1.10 (2.00)	0.680	+17% −13%
2	3	0.372 (0.40)	Not available	Not applicable	Not applicable	0.60 (0.80)	0.876	+3% −0%
2	4	436 (300)	0.0472 (0.015)	0.189 (0.054)	1.01E-3 (1.0E-3)	1.01E-3 (1.0E-3)	0.342	+48% −44%
3	5	1.09 (3.50)	104 (50)	1.51 (1.57)	21.0 (25.0)	0.0329 (1.00)	0.549	+33% −19%
3	6	0.036 (1.00)	0.727 (1.00)	0.719 (1.00)	0.686 (1.00)	0.381 (1.00)	0.517	+18% −12%
4	7	548 (9.02)	0.038 (1.25E-2)	Not available	720 (300)	0.20 (0.90)	0.753	+0% −9%
4	8	4.32E17 (1.56E15)	0.95 (0.857)	1120 (1500)	243 (350)	0.85 (0.80)	0.518	+19% −39%

$\overline{w}_i$ is defined as $\overline{w}_i = \sum_{j=1}^{5} w_{ij}\cdot F[(I_{ij}/I_{ij,0})]^{\alpha}$, where $\alpha = 1$ if $I_{ij}/I_{ij,0} \leq 1$, and $\alpha = -1$ if $I_{ij}/I_{ij,0} \geq 1$.

The pertinent FSDOB, referred to as a single cell of 500 kW, evaluated without incentives, is represented in Fig. 2, whereas Fig. 3 represents the corresponding position as a consequence of the following incentives: i) 50% of the initial investment as direct state incentives (204.500 €); ii) exemption from any form of taxation (VAT and other production taxes) on fossil fuels needed; iii) an additional contribution (of 163.380 €) as "green certificates".

Under such conditions, the fuel cell becomes competitive. At this stage, however, in order to justify such a high level of incentives, these must be compared with their associated Benefits.

7. Incentives and associated benefits

The induced Benefits due to an initial Investment I_0 (within its life time n) can be estimated on the basis of the Method of Barycentres. *Annual Economic Benefits* (*AEB*) can in fact be expressed as [11]:

$$AEB = \frac{I_0}{n}\cdot\left[I_{51,0}\cdot\sum_{i=1}^{8}\lambda_i\xi_i\overline{w}_i(1\pm\Delta\overline{w}_i)\right] \tag{8}$$

where λ_j are "scale coefficients" referred to axis 5 ("*Social–economic Benefits*"; $\lambda_5 = 1$), whereas ξ_i account for the specific orientation of

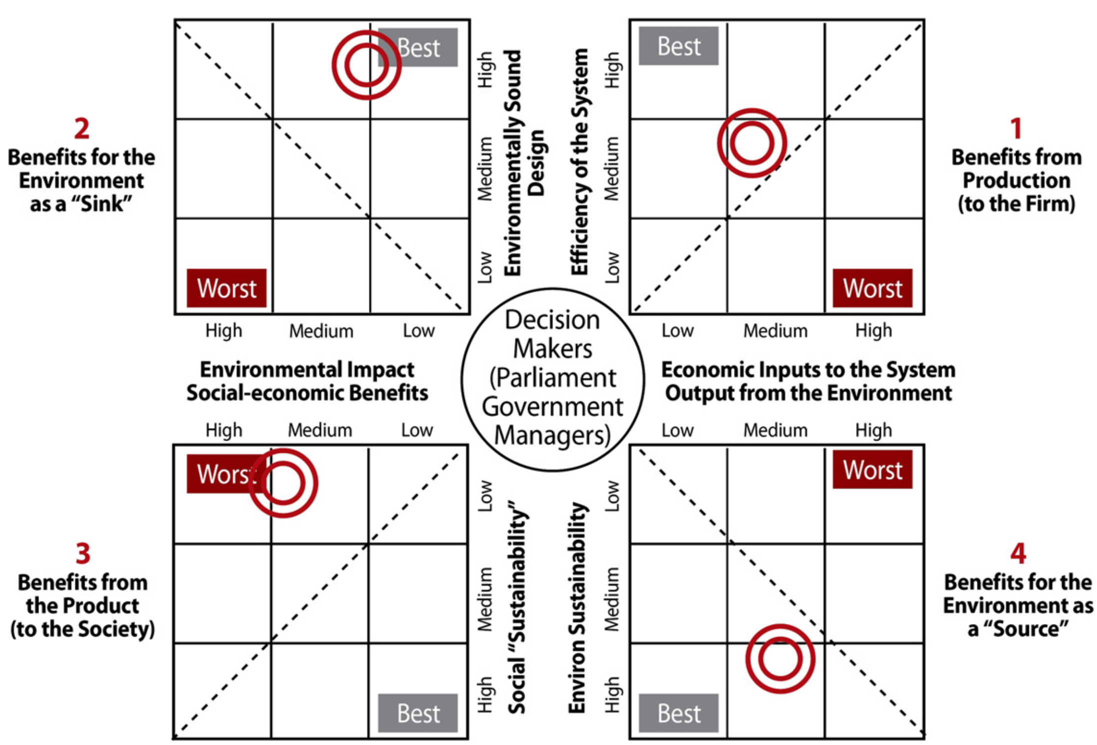

Fig. 2. Hydrogen fuel cells for stationary applications (without State incentives).

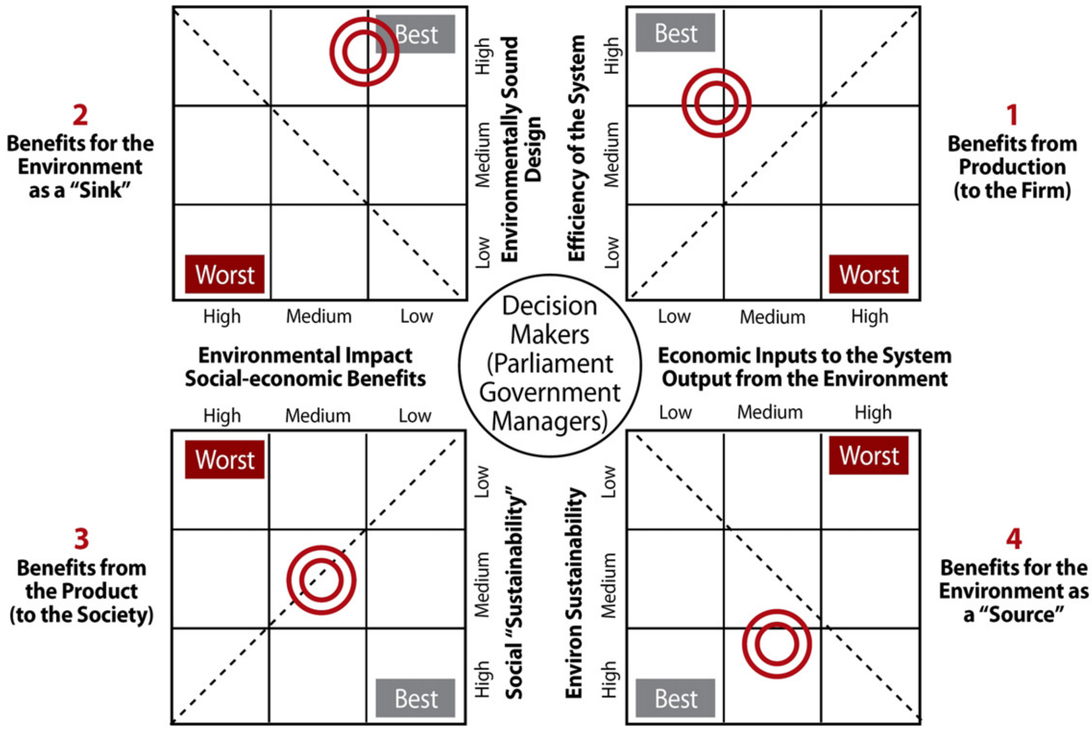

Fig. 3. Hydrogen fuel cells for stationary applications (with State incentives).

each axis. If we consider that Incentives (ΔI_0) are always a fraction (χ) of the Investment I_0, we obtain

$$AEB = \left[I_{51,0} \sum_{i=1}^{8} \lambda_i \xi_i \overline{w}_i (1 \pm \Delta \overline{w}_i) \right] \frac{\Delta I_0}{n\chi}. \tag{9}$$

In our case study (for $\chi = 0.5$, $I_{51,0} = 3.5$, $n = 5$, and for $\lambda_j = 1$) we have that

$$AEB \cong (5.1 \div 8.8) \cdot \Delta I_0. \tag{10}$$

This result represents the basic reason for the adoption of a Decision Making Process preferentially based on the estimated external *Benefits* (to be "remunerated" by means of appropriate Incentives) rather than on possible *damages* to be *internalized*. In addition, a Policy Maker should also consider the return on the Investment (about 50%) in terms of VAT and income tax due to all the related commercial activities finalized to realize the considered plant.

Such an evaluation procedure, however, could be considered as still being too subjective because of the presence of the "correlation" coefficients $\lambda_j (j \neq 5)$to be defined by the Decision Maker. This limit is mainly due to the fact that the four Sectors, although analyzed in Ordinal terms, are still considered as being substantially "independent" from each other. In spite of such an evident limitation, the evaluation of *AEB* so obtained is less "subjective" than it might seem at a first glance. This can be shown by comparing the previous results with those obtainable on the basis of a more general method. The previous limitation, in fact, can easily be overcome by directly passing to an Ordinal *dynamic* description, in terms of IDC, by means of which the analyzed System can be considered, from the very beginning, as being a real Whole. More precisely, as one sole Generative Process. In such a case the same values adopted in the previous analysis can still be used. However they will no longer be considered as the basic values for *AEB* evaluation, but only as a simple "cipher" of one sole Ordinal Relationship.

8. Ordinal benefits under dynamic conditions

Such an approach is favored by the fact that a dynamic Ordinal model of any complex system always presents an explicit solution in a *closed form* [12]. Consequently, the coefficients λ_j can be obtained on the basis of that relational structure which realizes the maximum Ordinality level of the system understood as a whole. Nonetheless, the same method can also be applied under steady state conditions because, as shown in Appendix, any process can be considered as the exit of a *Generative* Process.

As an ostensive example, we can consider the simplest case of Ordinal modeling. We can suppose, in fact, that the considered System has reached its *steady state* conditions as a consequence of a dynamic behavior in which each Sector is the real expression of an internal *generative co-operation* (thus represented by a "binary" function), amplified by all the Sectors which interact with one another as four consequential "duets". The system can thus be modeled as a "quartet" of "binary" functions (see Appendix, section B1). In such a case the proper variables to be considered are $x_i = \xi_i \overline{w}_i$ ($i = 1,2,\ldots 8$) (more than barycentres $\overline{w}_i$) so as to account for, at the same time, the specific orientation associated to each axis.

This aspect then suggested the development of a dynamic approach to the evaluation of AEB, in which such "coefficients" can be obtained as a direct consequence of an optimum Ordinal configuration of the system analyzed.

The System will be then represented as follows (a "quartet" of "binary functions")

$$\left[\begin{pmatrix} \lambda_1^* x_1 \\ \lambda_2^* x_2 \end{pmatrix}, \begin{pmatrix} \lambda_3^* x_3 \\ \lambda_4^* x_4 \end{pmatrix}, \begin{pmatrix} \lambda_5^* x_5 \\ \lambda_6^* x_6 \end{pmatrix}, \begin{pmatrix} \lambda_7^* x_7 \\ \lambda_8^* x_8 \end{pmatrix} \right] \tag{11}$$

in which coefficients λ_i^* (generally $\lambda_i^* \neq 1$) account for the fact that original values x_i ex$_{i+1}$ ($i = 1,3,5,7$) are not perfectly specular. This also means that quantities previously assumed as Indicators only partially represent the considered System, because they cannot be structured in the form of (supposed) binary functions. In this sense, the coefficients λ_i^* can be considered as representing a sort of "hidden variable", which satisfy a certain number of *adherent* conditions. The latter, in fact, cannot be assumed as being traditional "functional" relationships, because they represent the simple *cardinal reflex* of the original Ordinal Relation (a "quartet of binary functions") describing the System.

The conditions of specularity related to "binary functions" and "duet functions", respectively, require that

$$\lambda_i^* x_i = \lambda_{i+1}^* x_{i+1} \tag{12}$$

$$\lambda_i^* x_i = \lambda_{i+2}^* x_{i+2}, \tag{13}$$

where $i = 1,3,5$ (but also $i = 2,4,6$). Conditions (12) and (13) enable us to express all the coefficients λ_i^* in terms of the coefficient λ_5^* (which is strictly related to GDP)

$$\lambda_i^* = (x_5/x_i)\lambda_5^*. \tag{14}$$

The coefficient λ_i^* andλ_{i+1}^* ($i = 1,3,5,7$), however, should also be "consonant" with the *harmony conditions* (due to the "persistence of form") which characterize all Ordinal Systems (see Eq. (A.9) in Appendix)). We can thus recognize that, while (12) relates λ_5^* and λ_6^* in linear terms, Eq. (A.8) establishes another relationship in the form of a hyperbole (valid for each one of the four binary functions), such as

$$\Lambda_5^2 - \Lambda_6^2 + 2\Lambda_5 X_5 - 2\Lambda_6 X_6 + X_5^2 - X_6^2 = 0, \tag{15}$$

where

$$\Lambda_5 = \ln\lambda_5, \quad \Lambda_6 = \ln\lambda_6, \quad X_5 = \ln x_5, \quad X_6 = \ln x_6. \tag{16}$$

Equation (15), together with Eq. (12), enables us to define the value of λ_5^* and, consequently, the corresponding values of all the other coefficients λ_i^* (through Eq. (14)).

The general solution to problem indicates that: i) if the Process is originally characterized by all values x_i equal to each other, it can be assumed as being a perfect "quartet" of "binary functions" and, in such a case all λ_i^* are equal to 1; ii) if, on the contrary, the original values x_i differ from each other, the *harmony conditions* (see Eq. (15)) are perfectly respected when $\lambda_i^* = x_{i+1}$ ($i = 1,3,5,7$); iii) thus "duet" conditions (13) can be reformulated as follows

$$\psi_i^2 \cdot x_i x_{i+1} = \psi_{i+1}^2 \cdot x_{2i+1} x_{2i+2} \quad (i = 1,2,3); \tag{17}$$

iv) at this stage, if GDP is assumed as being the reference evaluation criterion, axis 5 (which corresponds to GDP) has no associated "hidden variable". The corresponding Annual Economic Benefits (Eq. (9)) can be thus evaluated as follows

$$\begin{aligned} AEB^* &= \left[I_{51,0}\sum_{i=1}^{8}\lambda_i\xi_i\overline{w}_i\right]\frac{\Delta I_0}{n\chi} = I_{51,0}\frac{\Delta I_0}{n\chi}\cdot\left[\frac{2}{\lambda_5^*}\cdot\sum_{i=1}^{4}\psi_i x_i x_{i+1}\right] \\ &= 4.94\cdot\Delta I_0; \end{aligned} \tag{18}$$

v) this result shows that Eq. (10) generally overestimates *AEB*. Eq. (9) then has preferably to be used in the form corresponding to its lower limit. This is simply due to the fact that the various indicators adopted, deriving from different disciplines (such as Energy analysis, Exergy analysis, Emergy accounting, Environmental impact assessment, Macroeconomic and Externality Evaluation, etc. (see Table 1)), after having been transformed into dimensionless numbers in the interval [0,1], simply represent the correct positioning of the considered system with respect to the best available technology pertaining to its specific class (whose reference values are represented in brackets in Table 2). In this way each axis actually represents a specific form of benefit, although the direct correlation between axes is left to Decision Maker's sensitivity. This procedure thus only leads to a *mean value* of *AEB* which, by itself, does not exactly reflect those relationships (such as harmony conditions) which require the adoption of values much more adherent to the global Ordinality. The latter, on the contrary, is a formal entity which does not suffer from the previous limitations. Its increase, in fact, although always due to the contribution of different Generative Processes, is not the result of a simple "sum" (and, thus, is not a "mean value"). The progressive increase in Ordinality certainly corresponds to a more complex System, but not in simply quantitative terms. In fact it represents, a much more *harmonious* structural over-organization of the System, the "complexity" of which is a simple cardinal "reflex" of the Ordinality level achieved by the System [5].

In spite of such considerations, which clearly show the basic difference between the two forms of evaluation, $AEB^*$are always greater than ΔI_0, so previous considerations (made in the case of steady state conditions) still remain valid; vi) such an evaluation procedure at the same time offers some useful indications to the Decision Maker in order to improve the structure of the FSDOB, even if the Process analyzed was not initially conceived in generative terms; vii) it also shows that, if the Plant/Process had already been conceived, from the very beginning, as a Generative Process (that is as a "quartet of binary sectors"), it would have been characterized, because its own nature, by *unitary* "correlation" coefficient ($\lambda_i^* \equiv 1$), by realizing, in such a case, the maximum corresponding (cardinal) benefits; viii) this also shows that only in such conditions does GDP really represent a real Indicator of both economic development and widely-diffused well-being; ix) in this case, in fact, any increase in GDP *reflects* (and it is also *a consequential "reflex"*) of the corresponding improvements in all the other considered Sectors; x) consequently, the most important aspect which emerges from such an evaluation procedure is that: the *real* optimum economic conditions (which, on the contrary, are usually assumed as being corresponding to Pareto's Optimum Efficiency Criterion) are exactly those which correspond, as an adherent reflex, to the *Maximum Ordinality* level achieved by the System, understood as a Whole. In other terms: Processes which tend to (and then realize) an ever higher level of Ordinality, also realize, in actual fact, the optimum corresponding cardinal (economic) benefits.

The level of Ordinality, in fact, does not depend on available resources. These can only have a specific influence on the sole Ordinal Stability of System. That is, the persistence of the System, in the long run, at the level of Ordinality achieved.

9. Conclusions

A simple case study was adopted to show the difference between Ordinal Benefits and economic (cardinal) benefits, in particular when the former are evaluated both in *static* and *dynamic* conditions. The basic difference between the two considered evaluations consists in the fact that: i) under (traditional) *static* conditions, the FSDOB method estimates, as economic "proxies", the Ordinal Externalities always associated to any given productive process, by adopting the mean value of the generative Transformities, although the latter are theoretically understood as a "cipher" of those External Benefits; ii) under dynamic conditions, on the contrary, the direct evaluation (by means of IDC) of the maximum Ordinality level achieved by the System, enables us to evaluate the optimum economic conditions (Incentives included) which correspond, as an adherent consequential reflex, to those optimal working conditions.

The latter evaluation procedure, however, as already anticipated (par. 8), can *also* be adopted under static conditions, by assuming a (preliminary) "equivalence" between the considered process and a corresponding Generative Process made up of a "quartet of binary functions". Such a more unitary description of the System, in fact, in spite of its (apparent) complexity, enables us to evaluate the correlation coefficients between the various Sectors which, in the most habitual cases, always remain affected by a certain degree of Decision Maker's subjectivity.

In essence: in the *traditional static* conditions, the FSDOB method (originally based on Emergy and Transformity concepts) converts Ordinal Benefits into (supposedly) equivalent economic terms understood as "proxies"; on the contrary, under both static and dynamic *Generative* conditions, the difference between Ordinal Benefits and economic benefits is clearly well-distinct. Optimal economic benefits, in fact, are not estimated as "proxies". They simply are the physical reflexes of the optimum Ordinal working conditions.

This clearly suggests that traditional economic maximization criteria (usually corresponding to Pareto Optimality) should preferably be replaced by the Ordinal Maximization Principle. The latter, in fact, enables the Decision Maker to recognize those optimal working conditions which realize the maximum Ordinality level of the System and, at the same time, to evaluate the corresponding optimum economic conditions (Investments, Benefits, Incentives, etc.) as a consequential adherent reflex.

Consequently, in the perspective the Maximum Ordinality Principle, the same "Incentives" can no longer be considered as a sort of "gift" to the Firm. They in fact constitute a form of "remuneration" of *Ordinal Externalities* (Benefits) that a Firm produces in favor of the Society and the Environment. The State, on the contrary, recuperates such "incentives" as a consequence of a *non-zero sum* circular process: either when these Ordinal Benefits are associated to activities never accounted for by GDP (even though they always represent benefits to Society), or when they are assumed as a reference "guide" to evaluate the optimum level of economic resources required. In the latter case the criterion here proposed also represents a valid contribution to the fundamental problem of defining both entity and extension of a possible public intervention in Economy but, above all, enables any Decision Maker to make decisions which are always, and even more, orientated toward the genesis of the *Maximum Ordinality Excess*.

Appendix. The incipient derivative and its basic properties

The analysis of Generative Processes under dynamic conditions suggests the introduction of a new concept of "derivative". This is because the same adoption of the traditional derivative (d/dt) is nothing but the formal reflex of three fundamental pre-assumptions when describing physical–biological–social systems: i) *efficient causality*; ii) *necessary logic*; iii) *functional relationships*. It is then evident that such an *aprioristic* perspective excludes, from its basic foundation, the possibility that any process output might ever show anything "extra", with respect to its corresponding input, as a consequence of the intrinsic (supposedly) *necessary, efficient and functional* dynamics of the system analyzed.

Consequently, such a theoretical approach will never see any "output excess", exactly because it has already excluded from the very beginning (but only aprioristically) that there might be "any". In this sense it is possible to say that such an approach describes all the phenomena as they were mere "mechanisms" (see par. 2). Generative Processes, on the contrary, suggest we think of a different form of "causality", precisely because their outputs always show something in "excess" with respect to their inputs. This "causality" may be termed as "generative" causality or "spring" causality or whatsoever. In all cases the basic concept is rather clear. In fact, any term adopted is simply finalized at indicating that it is worth supposing a form of "causality" which is capable of giving rise to something "extra" with respect to what it is usually foreseen (and expected) by the traditional approach.

The same happens for Logic. In fact, a different Logic is correspondently needed in order to contemplate the possibility of coming to conclusions much richer than their corresponding premises. This new form of Logic, in turn, could correspondently be termed as "adherent" Logic, because its conclusions are always faithfully conform to the premises. The former, however, could even be well-beyond what is strictly foreseen by the same premises when interpreted in strictly necessary terms.

As an adherent consequence of both previous concepts, the relationships between phenomena cannot be reduced to mere "functional" relationships between the corresponding *cardinal* quantities. In fact, they always "vehicle" something else, which leads us to term those relationships as "Ordinal" relationships. The term "Ordinal", which might appear as being simply adopted only to make a difference with respect to its corresponding "cardinal" concept, has in reality a much more profound meaning (as we will see later on).

At this stage we can clearly assert that the new concept of derivative is nothing but the adherent "translation", in formal terms, of the three new concepts: *generative Causality*, *adherent Logic*, *Ordinal relationships*. Such a new derivative was termed as "incipient" (or *prior* derivative) because it describes the processes in their *generating* activity or, preferably, it focuses on their pertinent outputs in their specific *act of being born*. Its mathematical definition is substantially based on the *reverse priority* of the order of the three elements that constitute the traditional definition:

$$\lim_{\Delta t\to 0}\frac{\Delta}{\Delta t}f(t) \tag{A.1}$$

that is: i) the concept of *function* (which is assumed to be a primary concept); ii) the *incremental ratio* (of the supposedly known function); iii) the operation of *limit* (referred to the result of the previous two steps). It is thus defined as follows (for further details see also [3]):

$$\frac{\tilde{d}^q}{\tilde{d}t^q}f(t) = \underset{\tilde{\Delta}t:0\to 0^+}{\tilde{Lim}} \circ \left(\frac{\tilde{\delta}\,\tilde{-}\,1}{\tilde{\Delta}t}\right)^q \circ f(t) \tag{A.2}$$

where: i) the symbol $\tilde{Lim}$ now represents a sort of "window" or "threshold" (=*Limen* in Latin), from which we observe and describe the considered phenomenon, whereas $\tilde{\Delta}t:0\to 0^+$ indicates not only the initial time of our registration, but also the proper "origin" (in its etymological sense) of something new which is being born; ii) the "*operator*" $\tilde{\delta}$ registers the variation of the property $f(t)$ analyzed, not only in terms of quantity, but also, and especially, in terms of Quality (as indicated by the symbol "tilde" specifically adopted); iii) thus the ratio ($\tilde{\delta}-1/\tilde{\Delta}t$) indicates not only a quantitative variation in time, but both the variation in Quality and quantity. That is, the *Generativity* of the considered process or, in other terms, the output "excess" (per unit time) characterized by both its Ordinality and its related cardinality; iv) the sequence of symbols in Eq. (A.2) is consequently interpreted according to a *direct priority* (from left to right); v) the sequence is also interpreted as a *generative inter-action* (represented by the symbol "$\circ$") between the three considered concepts; vi) the definition is valid for *any fractional number q*.

On the basis of such a definition, we can now show how it is possible to generalize, under dynamic conditions, the three basic Generative Processes pointed out by H.T. Odum [1,2]. To this aim, and for the sake of simplicity, we can always refer to Ordinal relationships represented by exponential functions (in the most general form $e^{\alpha(t)}$) because, as is well know, any function $f(t)$ can always be written as

$$f(t) = e^{\ln f(t)} = e^{\phi(t)} \tag{A.3}$$

A) Co-production process

This Process, schematically graphed in Fig. A.1, can formally be represented by means of a derivative of order 1/2. This derivative, in fact, gives rise to a "binary" function, that is: an output made up of two distinct entities, which however form *one sole thing*. This is equivalent to say that the two "by-products", precisely because generated by the same *unique* (Generative) Process, keep memory of their common and *in-divisible origin*, even if they may have, later on, completely different topological locations in time.

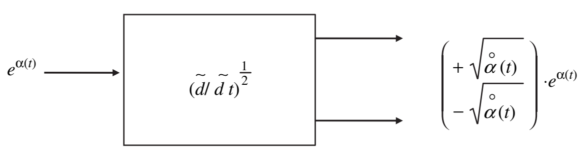

Fig. A.1. Representation of a co-production process.

The genesis of "binary" functions (from a co-production process) can formally be represented as:

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{\frac{1}{2}} e^{\alpha(t)} = \begin{pmatrix} +\sqrt{\mathring{\alpha}(t)} \\ -\sqrt{\mathring{\alpha}(t)} \end{pmatrix} \cdot e^{\alpha(t)} \tag{A.4}$$

where the order of the derivative 1/2 explicitly reminds us that the output generated is "1" sole entity, although made up of "2" parts. In other terms the output, when understood as a whole, is much more than the simple sum of its single parts. Said differently, the *uniqueness* of the Generative Process, recognized as being a specific property of a Co-generation Process, remains as being *in-divisible*, and thus also *ir-reducible* to the component parts.

A simple example of such a Generative Process can be represented by the Generation of two "twins", who always keep "trace" of their common Co-generation, not only at a genetic level, but also through several other characteristics.

Such an example can also be useful to illustrate that the corresponding equivalent of the above-mentioned genetic properties, can be represented, at a formal level, by the square root $\sqrt{\mathring{\alpha}(t)}$. This in fact represents a sort of "extraction" (on behalf of the derivative of order 1/2) of the "genetic properties" of the given Ordinal relationship $e^{\alpha(t)}$, whereas the symbols "$+/-$" characterize the corresponding distinct cardinalities (in reality, at a more general level of representation, these symbols will lose their algebraic sense, to assume a deeper meaning of internal relationships, and thus represented differently, for instance, as "$\oplus/\Theta$" [12]).

The concept of Co-generation Process, however, is not limited to living beings. This Generative Process, in fact, is also present in Classical Mechanics. Such a model, in fact, when adopted to describe the relationship between Sun and Mercury, understood as being generated by the same Laplace Nebula, is able to explain the famous Mercury's Precessions, by always keeping the same structure of Newtonian Laws, without any necessity of adopting General Relativity [12]. The same happens in Quantum Mechanics, where the same Co-generative model is able to interpret the famous (and still unexplained) "Entanglement" of two photons *co-generated* by the same process [12].

What's more (with respect to the specific finalities of this paper) it is also ever-present in Economics, precisely when the same Productive activity generates two or more "by-products".

B) Inter-Action Process

This Generative Process can easily be illustrated by considering first a single input Process (see Fig. A.2). In such a case the Process, modeled through the incipient derivative of Order 2, represents a *reinforcement* of the same input, so giving rise to a new entity which, however, is much more than the simple (cardinal) product of the original input by itself considered, and it can be thus represented as

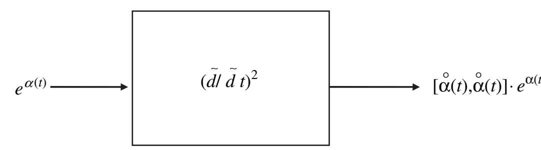

Fig. A.2. Formal representation of a "duet" process amplification.

This Process can be termed as "Generative" precisely because the two contributions not only reinforce each other, but are also unified in a new *one sole entity*. In other terms, they not only increase the cardinality of their joint action, but also generate an exceeding Quality, represented by the *uniqueness* and *irreducibility* of their co-operating activity, because *solidly* and *indissolubly* orientated in the same "direction". This is why the corresponding output can be termed as a "duet" function and represented, in formal terms, as follows

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{2} e^{\alpha(t)} = [\ \mathring{\alpha}(t),\ \mathring{\alpha}(t)] \cdot e^{\alpha(t)} \tag{A.5}$$

It is then easy to recognize that, only when such a Process is seen in mere *cardinal* terms, does the output reduce to the traditional result of a *scalar* product between the two quantities $\mathring{\alpha}(t)$, by giving

$$\left(\frac{d}{dt}\right)^{2} e^{\alpha(t)} = [\ \mathring{\alpha}(t) \cdot \ \mathring{\alpha}(t)] \cdot e^{\alpha(t)} = [\ \mathring{\alpha}(t)]^2 \cdot e^{\alpha(t)}. \tag{A.6}$$

In such a case, in fact, the process is coherently described by means of the traditional derivative (see Eq. (A.6)), which, as repeatedly asserted, "filters" any form of Ordinality.

B1) The inter-Action Process in its proper sense

The Inter-Action Process, in its proper definition, manifests its true essence in the presence of (at least) two distinct inputs and it can be thus represented as in Fig. A.3.

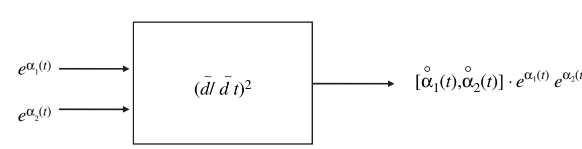

Fig. A.3. Representation of an Inter-Action Process.

where the "duet" $[\ \mathring{\alpha}_1(t),\ \mathring{\alpha}_2(t)]$ now stands for the Logic "and": $[\ \mathring{\alpha}_1(t),\ \mathring{\alpha}_2(t)] \wedge [\ \mathring{\alpha}_2(t),\ \mathring{\alpha}_1(t)]$.

It is thus characterized by the total absence of any form of internal reciprocal priority.

It is also worth mentioning that the Inter-Action Process is very frequently associated to a Co-generation Process. In such a case we can also speak of an Inter-Action Process characterized by a "subjacent" Co-generation Process (with its associated "binary" function). The Process can be then characterized by a derivative of Order 2/2 and thus represented as in Fig. A.4.

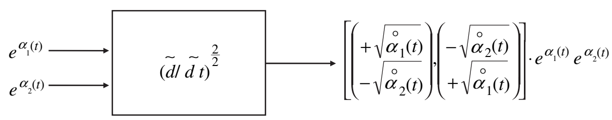

Fig. A.4. Representation of an Inter-Action Process (with a "subjacent" Co-generation).

In such a case the two inputs not only contribute to a reciprocal reinforcement, but are also reciprocally coupled in the form of a "binary" function. In addition, such a coupling, is further enhanced by the inter-exchange (and successive coupling) of the specific "genetic" properties of the input Ordinal functions (see $\sqrt{\mathring{\alpha}_1(t)}$ and $\sqrt{\mathring{\alpha}_1(t)}$, respectively). The Process thus gives rise to a "duet-binary" function:

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{\frac{2}{2}}\left[e^{\alpha_1(t)}e^{\alpha_2(t)}\right] = \left[\begin{pmatrix}+\sqrt{\mathring{\alpha}_1(t)}\\-\sqrt{\mathring{\alpha}_2(t)}\end{pmatrix},\begin{pmatrix}-\sqrt{\mathring{\alpha}_2(t)}\\+\sqrt{\mathring{\alpha}_1(t)}\end{pmatrix}\right]\cdot e^{\alpha_1(t)}e^{\alpha_2(t)}. \tag{A.7}$$

A significant example of this Generative Process can be represented by the generation of a living being. The formal expression (A.7), in fact, would be a preliminary representation of the recomposition of a *completely new* couple of chromosomes by starting from one chromosome pertaining to the father and the other pertaining the mother. Evidently, the Process is here extremely simplified. In fact, in the human case (for instance) we should have to consider 23 couples of chromosomes deriving from the father and 23 from the mother, respectively, which give rise to a *completely new* human being, characterize by 46 new couples of chromosomes.

On the basis of such premises we can now show how the four Sectors, initially thought of as a sequence of four "duets" interacting each other, can be represented as a quartet of "binary" functions. This can simply be obtained on the basis of the fact that the initial four "duets" would properly originate a "quartet of duets". In addition, by taking into account that each "duet" generally shows a subjacent co-generation activity, the more general model would be represented by a "quartet of duet-binary functions". However, the choice of considering the four Sectors according to a *specific sequence* (from 1 to 4), allows us to introduce some simplifications. In fact the specific properties of duets (see Eq. (13)) and the fact they are always considered in a *prefixed* sequence, allows us to reduce (by a sort of "contraction") the Ordinality of the duets, by always keeping their specific cardinality (see Eq. (A.6)). This procedure transforms the resulting structure into a quartet of *sequential* binary functions, very similar to the form (11) and, at the same time, makes the original structure more directly comparable with the one adopted in steady state conditions.

Such a *decomposition/reduction* procedure is of fundamental importance in any Ordinal analysis, exactly because it represents the *only way* to make possible a comparison with thecorresponding traditional cardinal analysis, which does not consider *any* form of Ordinality. Nonetheless, the same procedure, although "simplified" in order to get such a *preliminary* result, contemporaneously shows how it is possible to represent the considered system in terms of progressively increasing levels of Ordinality.

C) Ordinal feed-back

This Process can easily be illustrated on the basis of the Inter-Action Process, by assuming that the Ordinal output of the Process contributes, together with the input, to its same genesis (see Fig. A.5).

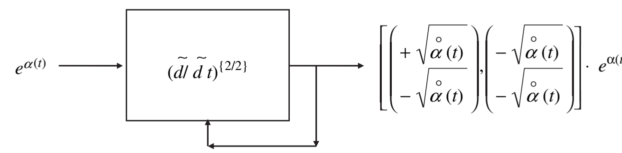

Fig. A.5. Representation of an ordinal feed-back process.

In such a case the output represents a perfect specular reproduction of the input, although at a *higher* Ordinality level. This is why the derivative of Order {2/2} is specifically represented in brackets: to expressly point out such a specific *harmonic consonance* between the input and the output of the Ordinal Feed-back Process, which can be represented in formal terms as follows

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{\{2/2\}} e^{\alpha(t)} = \left[\begin{pmatrix}+\sqrt{\mathring{\alpha}(t)}\\-\sqrt{\mathring{\alpha}(t)}\end{pmatrix},\begin{pmatrix}-\sqrt{\mathring{\alpha}(t)}\\-\sqrt{\mathring{\alpha}(t)}\end{pmatrix}\right]\cdot e^{\alpha(t)}. \tag{A.8}$$

At this stage, Eqs. (A.4), (A.5) and (A.8) represent the formal generalization of the Rules of Emergy Algebra corresponding to the three mentioned Generative Processes. They also show, in each case, the pertinent genesis of an *excess of Quality*. In fact, Coproduction Transformity is now replaced by the *Ordinality* 1/2 (that is the power of the derivative $(\tilde{d}/\tilde{d}t)$ understood in an Ordinal sense). The same happens for the Inter-action Process, now represented by the incipient derivative of order 2. Finally, the most elementary Feed-back Process is represented by the incipient derivative of order {2/2}, understood as a unique formal entity.

In this respect it is worth noting that such an Ordinality {2/2} does not correspond to the cardinal value of "1", nor does it correspond to 2/2, because the Ordinal Feed-back Process is not reducible to a simple "combination" of the two previous Processes. The same Eq. (A.8) clearly expresses, by itself, such an "excess" of Ordinality with respect to Eq. (A.7). The former in fact represents an "excess" in the *interior harmony relationships* due to *the persistence of form* (see later on) which intimately relates to each other the four distinct elementary functions which appear on its right hand side, now organized in *one sole* ir-reducible structure.

This can be easily understood by the fact that, in the most general case, the *incipient* derivative of order m/n is given by

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{m/n} e^{\alpha(t)} = [\,\mathring{\alpha}(t)]^{(m/n)}\cdot e^{\alpha(t)} \tag{A.9}$$

where $\mathring{\alpha}(t)$ represents the first-order *incipient* derivative of the function $\alpha(t)$ and $[\,\mathring{\alpha}(t)]^{(m/n)}$ represents a multiple binary–duet function of Ordinality (m/n).

The little circle characterizing the incipient derivative $\mathring{\alpha}(t)$ was evidently chosen in analogy to classical Newton's "dot" notation, usually adopted to indicate a first-order derivative.

The different symbology is here justified by the fact that the former should now remind us the *conceptual difference* between the incipient derivative and the traditional one. In fact, even if $\mathring{\alpha}(t)$ and $\dot{\alpha}(t)$ coincide from a pure cardinal point of view, they are, on the contrary, radically different from a *Generative* (and also Ordinal) point of view. The former, in fact, represents the specific exit of a *Generative* Process, whereas the latter is always understood as the result of a *necessary* process (thought of as being a "mechanism" or a set of "mechanisms").

Moreover, such a purely quantitative coincidence is strictly valid only for $n = 1$. In fact, in the general case of an Ordinal exponent (m/n), Eq. (A.9) shows all the significance of its output Ordinal structure (in terms of multiple binary–duet functions) and, at the same time, the deep difference with respect to the corresponding *cardinal* fractional derivative of order m/n usually considered in Literature [13].

In addition, the right hand side of Eq. (A.9) reveals an extremely important property: a sort of "persistence of form". This exactly because it represents an "adherent" consequence of a Generative Process, characterized by specific generation modalities. In other words, any "generating process" (modeled by the left hand side of Eq. (A.9)) gives origin to an Ordinal output (characterized by the Ordinality (m/n)) which corresponds to a multiple structure functions (described by the right hand side of Eq. (A.9)). These functions are similar to harmonic evolutions always in "resonance" (as in a "musical chord") with the original function and at the same time with each other, and they reach their *maximum harmony* in the case of a perfect Ordinal Feed-back $\{n/n\}$.

Such resonance relationships (whose number and typology are defined by the Ordinality (m/n)), when formalized in explicit terms, represent the afore-mentioned *interior harmony relationships*. These in fact express particular "coupling conditions" between integer and fractional derivatives [6]. For example

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{(1/2)} f(t)\circ\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{(1/2)} f(t) = f(t)\circ\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{(2/2)} f(t) = \left(\frac{\tilde{d}}{\tilde{d}t}\right)^{(2/2)} f(t)\circ f(t), \tag{A.10}$$

which is always valid, also under *steady state* conditions

$$\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{(1/2)} f(0)\circ\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{(1/2)} f(0) = f(0)\circ\left(\frac{\tilde{d}}{\tilde{d}t}\right)^{(2/2)} f(0) = \left(\frac{\tilde{d}}{\tilde{d}t}\right)^{(2/2)} f(0)\circ f(0), \tag{A.11}$$

and for any function $f(t)$. In fact, all the above-mentioned properties, previously illustrated with reference to the simple exponential function $e^{\alpha(t)}$, can be easily generalized to any given function $f(t)$ on the basis of Eq. (A.3).

Consequently, the concepts of Co-production, Inter-action and Feed-back, initially illustrated by means of Eqs. (A.4), (A.5) and (A.8), can always be adopted to describe *any* dynamic Generative Process, however complex it is. This also due to the fact that, while the right hand sides of Eqs. (A.4), (A.5) and (A.8) represent the Ordinal structure of Co-production, Inter-action and Feed-back Processes, respectively, the corresponding left hand sides have an *identical structure*, always in the form $(\tilde{d}/\tilde{d}t)^q$, where q is a rational number which assumes the values of 1/2, 2 and {2/2}, respectively. This means that all Generative Processes are characterized by the same "subjacent" Generativity, which, however, can assume different forms, according to the Ordinality q. That is, a Generativity of Ordinal nature, because characterized by a specific Ordinality since the very beginning of the Process. This enables us to assert that Generative Transformity (generally and properly defined under steady state conditions) is nothing but a reflex of an Ordinal Generativity.

In fact, it is worth pointing out that all such properties are also valid under "steady-state" conditions. This is due to the fact that any Process, even in such conditions, is always the exit of a Generative activity. Thus any "constant" value describing its "steady-state" conditions has always the same form as (A.3), that is

$$f(t) = const = e^{\ln const} = e^{\phi(t)} \tag{A.12}$$

where $\phi(t)$ has to be adherently and properly thought of as

$$\phi(t) = (\ln const)\cdot\tilde{1}(t) = (\ln const)\cdot\int_0^{\tilde{1}}\tilde{\delta}(\tau)\cdot d\tau, \tag{A.13}$$

where $\tilde{1}(t)$corresponds to the Heaviside function (for $t \geq 0^+$), and $\tilde{\delta}(t)$ is the incipient Dirac Delta function, which coincides with the traditional Delta function only for $t \geq 0^+$.

Such a more general modeling capacity of *incipient* derivatives, associated with the afore-mentioned property that *any* Ordinal dynamic model always presents an explicit solution in a *closed form* [5], confers to the Incipient Differential Calculus much wider potentialities with respect to the Traditional Differential Calculus [5]. This is also confirmed by the fact that such a new mathematical approach led us to the solution of the famous "Three-body Problem" [12], which, on the other hand, played a very important role in Neo-Classical Economics too.

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