

## Uncertainty characterization for emergy values

Wesley W. Ingwersen\*

Center for Environmental Policy, Department of Environmental Engineering Sciences, University of Florida, P.O. Box 116350, Gainesville, FL 32611-6350, United States

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### ABSTRACT

While statistical estimation of uncertainty has not typically accompanied published emergy values, as with any other quantitative model, uncertainty is embedded in these values, and lack of uncertainty characterization makes their accuracy not only opaque, it also prevents the use of emergy values in statistical tests of hypotheses. This paper first attempts to describe sources of uncertainty in unit emergy values (UEVs) and presents a framework for estimating this uncertainty with analytical and stochastic models, with model choices dependent upon on how the UEV is calculated and what kind of uncertainties are quantified. The analytical model can incorporate a broader spectrum of uncertainty types than the stochastic model, including model and scenario uncertainty, which may be significant in emergy models, but is only appropriate for the most basic of emergy calculations. Although less comprehensive in its incorporation of uncertainty, the proposed stochastic method is suitable for all types of UEVs. The distributions of unit emergy values approximate the lognormal distribution with variations depending on the types of uncertainty quantified as well as the way the UEVs are calculated. While both methods of estimating uncertainty in UEVs have their limitations in their presented stage of development, this paper provides methods for incorporating uncertainty into emergy, and demonstrates how this can be depicted and propagated so that it can be used in future emergy analyses and permit emergy to be more readily incorporated into other methods of environmental assessment, such as LCA.

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### 1. Introduction

Emergy, a measure of energy used in making a product extending back to the work of nature in generating the raw resources used (Odum, 1996), arises from general systems theory and has been applied to ecosystems as well as to human-dominated systems to address a scientific questions at many levels, from the understanding ecosystem dynamics (Brown et al., 2006) to studies of modern urban metabolism and sustainability (Zhang et al., 2009). Emergy, or one any the many indicators derived from it (Brown and Ulgiati, 1997), is not an empirical property of an object, but an estimation of embodied energy based on a relevant collection of empirical data from the systems underlying an object, as well as rules and theoretical assumptions, and therefore cannot be directly measured. In the process of emergy evaluation, especially due to its extensive and ambitious scope, the emergy in a object is estimated in the presence of numerical uncertainty, which arises in all steps and from all sources used in the evaluation process.

The proximate motivation for development of this model was for use of emergy as an indicator within a life cycle assessment (LCA) to provide information regarding the energy appropriated from the

environment during the life cycle of a product. The advantages of using emergy in an LCA framework are delineated and demonstrated through an example of a gold mining (Ingwersen, under review). The incorporation of uncertainty in LCA results is commonplace and furthermore prerequisite to using results to make comparative assertions that are disclosed to the public (ISO 14044: 2006).

But the utility of uncertainty values for emergy is not only restricted to emergy used along with other environmental assessment methodologies; uncertainty characterization of emergy values has been of increasing interest and in some cases begun to be described by emergy practitioners (Bastianoni et al., 2009) for use in traditional emergy evaluations. Herein lies the ultimate motivation for this manuscript, which is to provide an initial framework for characterization of uncertainty of unit emergy values (UEVs), or inventory unit-to-emergy conversions, which can be applied or improved upon to characterize UEVs for any application, whether they be original emergy calculations or drawn upon from previous evaluations.

#### 1.1. Sources of uncertainty in UEVs

Uncertainty in UEVs may exist on numerous levels. Classification of uncertainty is helpful for identification of these sources of uncertainty, and for formal description of uncertainty in a repli-

\* Tel.: +1 352 392 2425; fax: +1 352 392 3624.

E-mail address: [wwi@ufl.edu](mailto:wwi@ufl.edu).

**Table 1**  
Elements of uncertainty in the UEV of lead in the ground.

Uncertainty type	Definition	Example	Explanation
Parameter	Uncertainty in a parameter used in the model	Flux of continental crust = .0024 cm/year	Global average number. A more recent number is .003 cm/year (Scholl and von Huene, 2004)
Model	Uncertainty regarding which model used to make estimations is appropriate	See model for minerals in Table 2	Variation exists between this model and others proposed for minerals
Scenario	Uncertainty regarding the fit of model parameters to a given geographical, temporal, or technological context	Variation in enrichment ratio based on deposit type	Assumption that the energy in all minerals of a given form is equal

cable fashion. The classification scheme defined by the US EPA defines three uncertainty types: parameter, scenario, and model uncertainty (Lloyd and Ries, 2007). This scheme is co-opted here to represent the uncertainty types associated with UEVs. These uncertainty types are defined in Table 1 using the example of the UEV for lead in the ground.

There are additional elements of uncertainty in the adoption of UEVs from previous analyses. These occur due to the following:

- Incorporation of UEVs from sources without documented methods.
- Errors in use of significant figures.
- Inclusion of UEVs with different inventory items (e.g. with or without labor & services).
- Calculation errors in the evaluation.
- Conflicts in global baseline underlying UEVs, which may be propagated unwittingly.
- Use of a UEV for an inappropriate product or process.

These bulleted errors are due to random calculation error, human error, and methodological discrepancy, which is not well-suited to formal characterization, and can be better addressed with more transparent and uniform methodology and critical review. But uncertainty and variability in parameters, models, and scenarios can theoretically be quantified.

## 1.2. Models for describing uncertainty in lognormal distributions

Different components of uncertainty in a model must be combined to estimate total uncertainty in the result. These component uncertainties may originate from uncertainty in model parameters. In multiple parameter models, such as emergy formula models, each parameter has its own characteristic uncertainty. Uncertainty in environmental variables is often assumed to be normal, although Limpert et al. (2001) presents evidence that lognormal distributions are more versatile in application and may be more appropriate for parameters in many environmental disciplines. This distribution is increasingly used to characterize data on process inputs used in life cycle assessments (Huijbregts et al., 2003; Frischknecht et al., 2007a,b).

A spread of lognormal variable can be described by a factor that relates the median value to the tails of its distribution. Slob (1994) defines this value as the dispersion factor,  $k$ , but it is also known as the geometric variance,  $\sigma_{\text{geo}}^2$ :

$$\sigma_{\text{geo of } a}^2 = e^{1.96\sqrt{\ln \omega_a}} \quad (1)$$

$$\omega_a = 1 + \left(\frac{\sigma_a}{\mu_a}\right)^2 \quad (2)$$

where  $\sigma_{\text{geo}}^2$  for variable  $a$  is a function of  $\omega_a$  (Eq. (1)),<sup>1</sup> which a simple transformation of the coefficient of variation (Eq. (2)),<sup>2</sup> where  $\sigma_a$  is the sample standard deviation of variable  $a$  and  $\mu_a$  is the sample mean. This can be applied to positive, normal variables with certain advantages, because parameters for describing lognormal distributions result in positive confidence intervals, and the lognormal distribution approximates the normal distribution with low dispersion factor values.

The geometric variance,  $\sigma_{\text{geo}}^2$ , ( $k \approx \sigma_{\text{geo}}^2$ ) is a symmetrical measure of the spread between the median, also known as the geometric mean,  $\mu_{\text{geo}}$ , and the tails of the 95.5% (henceforth 95%) confidence interval (Eq. (3)).

$$CI_{95} = \mu_{\text{geo}}(x \div) \sigma_{\text{geo}}^2 \quad (3)$$

The symbol '(x÷)' represents 'times or divided by'. The geometric mean for variable  $a$  may be defined as in the following expression (Eq. (4)):

$$\mu_{\text{geo}} = \frac{\mu_a}{\sqrt{\omega_a}} \quad (4)$$

The confidence interval describes the uncertainty surrounding a lognormal variable, but not for a formula model that is a combination of multiplication or division of each of these variables. The uncertainty of each model parameter has to be propagated to estimate a total parameter uncertainty. This can be done with Eq. (5):

$$\sigma_{\text{geo of model}}^2 = e^{\sqrt{\ln(\sigma_{\text{geo of } a}^2)^2 + \ln(\sigma_{\text{geo of } b}^2)^2 + \dots + \ln(\sigma_{\text{geo of } z}^2)^2}} \quad (5)$$

where  $a, b \dots z$  are references to parameters of a multiplicative model  $y$  of the form  $y = \Pi a \dots z$ . Note that parameter uncertainties are not simply summed together, which would overestimate uncertainty. This solution (Eq. (5)) is valid under the assumption that each model parameter is independent and lognormally distributed.

Describing the confidence interval requires the median, or geometric mean, as well as the geometric variance. The geometric mean of a model can be estimated first by estimating the model CV (Eq. (6)) and then with a variation of Eq. (4) (Eq. (7)).<sup>3</sup>

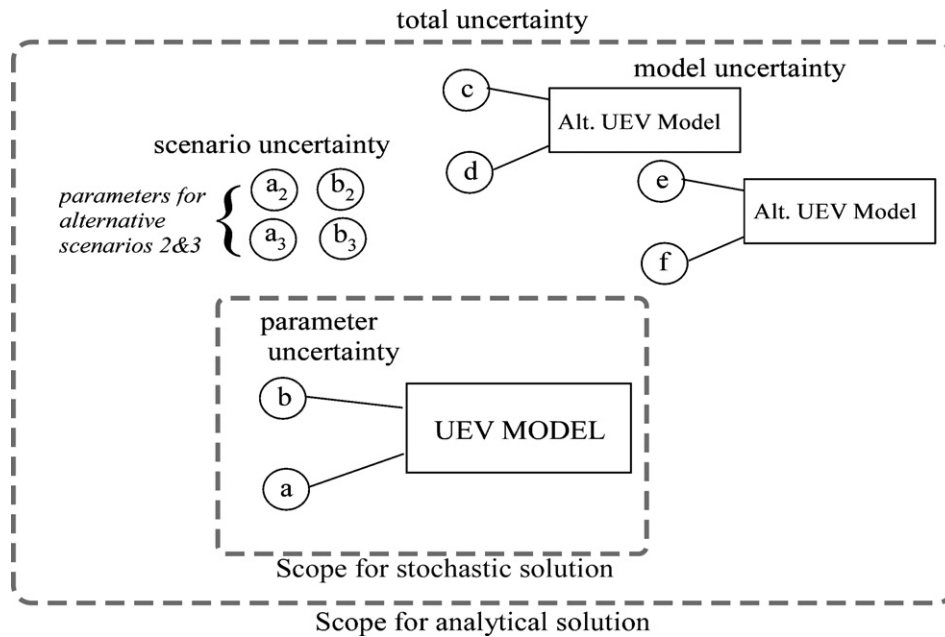
$$CV_{\text{model}} = \sqrt{e^{(\ln(\sigma_{\text{geo of model}}^2)^2 / 1.96^2) - 1}} \quad (6)$$

$$\mu_{\text{geo of model}} = \frac{\mu_{\text{model}}}{\sqrt{1 + CV_{\text{model}}^2}} \quad (7)$$

<sup>1</sup> Eq. (1) adapted from Slob (1994).

<sup>2</sup> Eqs. (2)–(4) adapted from Limpert et al. (2001).

<sup>3</sup> Eqs. (5)–(7) adapted from Slob (1994).



**Fig. 1.** Conceptual approach to modeling uncertainty. The parameter uncertainty consists of uncertainty and variability in the parameters used to estimate the UEV; the scenario uncertainty consists of the uncertainty arising from use of parameter values for different geographic or technological scenarios; the model uncertainty from different models. Only the proposed analytical solution incorporates scenario and model uncertainty to estimate total uncertainty.

## 2. Models for uncertainty in UEVs

Numerous methods exist for computing unit energy values,<sup>4</sup> but for uncertainty estimation, it is important to distinguish between them according to a fundamental difference in the way UEVs are calculated: the formula vs. the table-form model. The formula model is used for estimation of energy in raw materials, such as minerals, fossil fuels and water sources (the UEV in Table 1 is of this form). The traditional table-form evaluation procedure is typically used for ecosystem products and products of human activities. Formula models are generally multiplicative models using estimates of various biophysical flows and storages in the biosphere as parameters. In order to quantify variability within a formula model, such as an energy calculation, the result distribution needs to be known or at least predicted. Model parameters are generally positive values multiplied to generate the UEVs. Such multiplicative formulas have been shown to lead to results approximating a log-normal distribution (Limpert et al., 2001; Hill and Holst, 2001). Therefore it would be logical to assume that UEVs calculated in this manner are distributed lognormally.

The model geometric mean and variance (Eqs. (5) and (7)), used in conjunction, offer an analytical solution for estimating uncertainty for formula-type unit energy values, with some built in assumptions, foremost being that the model parameters have a common lognormal distribution. For models with parameters of mixed and unknown distributions and large coefficients a variation, a common method for estimating uncertainty is to simulate a model distribution using a stochastic method such as Monte Carlo, and estimate uncertainty based on the model distribution's confidence interval (Rai and Krewski, 1998). A notable drawback of a stochastic simulation method is that the results obtained have some variability in themselves, which, however, can be reduced by increasing the number of iterations.

Table-form UEV calculations would be more accurately described as sum products, where UEVs of inputs contributing to the total energy in an item of interest are multiplied by the quantities of each input to get energies in those inputs, and the energy in each input is then added together to get the total energy in the item of interest. This hybrid form operation is not readily amenable to an analytical solution (Rai and Krewski, 1998). In the absence of a readily-available analytical model for this type of UEV, a Monte Carlo model may be adopted for modeling UEV uncertainty for table-form calculations.

Fig. 1 provides a conceptual overview of the proposed uncertainty model. The analytical solution is used to model all quantifiable sources of uncertainty (parameter, model, and scenario) while the Monte Carlo model is used only to estimate total parameter uncertainty.

### 2.1. Modeling procedure and analysis

First the geometric variance and medians of five formula-type UEVs are estimated with the analytical solution to describe the type of variability and distribution of some commonly used UEVs, breaking down the uncertainty into the three classes described. Parameter uncertainty for these same UEVs is then also estimated with the stochastic model, along with two table-form UEVs. The modeling results are cross compared. As the distribution of UEVs has not previously been described, the resulting distributions from the stochastic model are tested to see how closely they fit traditional lognormal and normal distributions, as well as a hybrid of the two. In the process of this analysis a means of reporting UEV uncertainty for future incorporation and interpretation of uncertainty is described.

Uncertainty was estimated for five formula-type UEVs: lead, iron, petroleum, groundwater, and labor. These UEVs were chosen because they represent categories of inputs from the biosphere (labor excepted) – scarce and abundant minerals, petroleum, water, and human input – that form the basis of many product life cycles.

Models for calculating each UEV are presented in Table 2 along with their sources. Parameter uncertainty was estimated as follows: ranges of values or multiple values from distinct sources

<sup>4</sup> See Odum (1996) for procedure for calculating UEVs, which are also known as "transformities" when the denominator is an energy unit, or "specific energy" when the denominator is a mass unit.

**Table 2**  
Unit energy value models used for parameter uncertainty calculations.

Category	Model	Source
Minerals	UEV <sub>mineral</sub> = Enrichment Ratio × Land Cycle UEV (sej/g)	Cohen et al. (2008)
	Enrichment Ratio = (ore grade cutoff, %)/(crustal concentration, ppm)/(1E6) <sup>a</sup>	Cohen et al. (2008)
	Land Cycle UEV (sej/g) = (emergy base line, 15.83E + 24 sej/yr)/(crustal turnover, cm/yr)(density of crust, g/cm <sup>3</sup> )(crustal area, cm <sup>2</sup> )	Odum (1996)
Petroleum	UEV <sub>petro</sub> (sej/J) = (1.68 <sup>b</sup> × emergy of kerogen, sej/J)/(C content, %)/((conversion of kerogen to petroleum, fraction)(enthalpy of petroleum, 4.19E4 J/g))	Bastianoni et al. (2005)
	UEV <sub>carbon</sub> in kerogen (sej/g) = (emergy of C in phytoplankton, sej/g)/(conversion to kerogen, fraction)	Bastianoni et al. (2005)
	UEV <sub>carbon</sub> in phytoplankton, sej/g = (phytoplankton UEV, sej/J) × (phytoplankton Gibbs Energy, 1.78E4 J/g)/(phytoplankton fraction C)	Bastianoni et al. (2005)
Groundwater	UEV <sub>groundwater</sub> (sej/g) = (emergy base line, 15.83E + 24 sej/yr)/(Annual flux, g/yr)	Buenfil (2001)
	Annual flux (g/yr) = ((precipitation on land, mm/yr)/(1E6 mm/km <sup>2</sup> ) × (land area, km <sup>2</sup> ) × (infiltration rate, %) × (1E12 L/km <sup>3</sup> )(1000 g/L)	Buenfil (2001)
Labor	Total annual emery use model. UEV <sub>labor</sub> (sej/J) = ((emergy use) <sup>c</sup> /(population) × (per capita calorie intake, kcal/day)(365 days/yr)(4184 J/kcal))	Odum (1996)

<sup>a</sup> Omitted when concentration is reported in %.

<sup>b</sup> Included for conversion from global emery baseline of 9.44E + 24 to 15.83E + 24 sej/yr.

<sup>c</sup> Emery use for global estimate was 1.61E + 26 sej/yr, total emery use of the world's nations (Cohen et al., 2008).

when available were taken from the literature for each model parameter. The mean and sample standard deviation for each model parameter was calculated. With this value, the uncertainty factor,  $\omega$ , corresponding to each parameter was calculated with Eq. (2). The UEV *parameter uncertainty* was then estimated for the combined parameter uncertainty factors with Eq. (4).

*Model and/or scenario uncertainty* was incorporated by estimation of separate uncertainty factors for these types of uncertainty. When multiple models existed for a UEV, the average and sample standard deviation of the UEVs produced by different models were calculated. Model uncertainty was estimated for lead, iron, petroleum and water. When models exist for UEVs which are specific to a set of conditions but for which those conditions are unknown in the adoption of a UEV, scenario uncertainty can be included. For instance if labor is an input in a process, but the country in which the labor takes place is undefined, there is scenario uncertainty which includes the variability of the emery in the labor depending on which country it comes from. Two scenario uncertainties were estimated for labor UEVs (one for US labor and one for world labor) for purposes of example. Parameter along with either model or scenario uncertainty were combined for an estimate of *total uncertainty* by combining the uncertainty factors for each parameter and for scenario and/or model uncertainty according to Eq. (5). This can be summarized as:

$$\text{total uncertainty} = \text{parameter uncertainty} + \text{model uncertainty} + \text{scenario uncertainty} \quad (8)$$

**Table 3**  
Analytical uncertainty estimation for lead UEV, in ground.

No.	Parameters	$\mu$	$\sigma$	$\sigma_{\text{geo}}^2$
1	Crustal concentration (ppm)	1.50E+01	1.41	1.20
2	Ore grade (fraction)	0.06	0.03	2.25
3	Crustal turnover (cm/yr)	2.88E-03	6.77E-04	1.58
4	Density of crust (g/cm <sup>3</sup> )	2.72	0.04	1.03
5	Crustal area (cm <sup>2</sup> )	1.48E+18	2.1E+16	1.03
Models				
6	Alternate Model UEVs	4.52E+11	7.25E+11	9.12
Summary				
	Unit energy value, $\mu$ (sej/g)	5.46E+12		
	Parameter Uncertainty Range (No. 1–5), $\mu_{\text{geo}}$ (sej/g) ( $x \div$ ) $\sigma_{\text{geo}}^2$	4.85E+12	( $x \div$ )	2.59
	Total Uncertainty Range (No. 1–6), $\mu_{\text{geo}}$ (sej/g) ( $x \div$ ) $\sigma_{\text{geo}}^2$	2.57E+12	( $x \div$ )	11.09

Sources: 1. Odum (1996); Thornton and Brush (2001), 2. Gabby (2007), 3. Odum (1996); Scholl and von Huene (2004), 4. Australian Museum (2007); Odum (1996), 5. UNSTAT (2006); Taylor and McLennan (1985); Odum (1996), 6. ER method and Abundance-Price Methods (Cohen et al., 2008).

In order to compare the consistency of the analytical solution for the median and geometric variance with the confidence interval generated by the simulation, stochastic simulation models for the lead, iron, water, and labor UEV calculations were run. A Monte Carlo simulation was scripted in R 2.6.2 statistical software ©(R Development Core Team, 2008) to calculate each UEV 100 times using a randomly selected set of parameters. Randomized parameters were created with a random function using the sample standard deviation and means of each parameter. The parameters were assumed to be log-normally distributed.

The mean and standard deviations of the log-form of each parameter were used to create variables with a lognormal distribution, for which the following equations (Eqs. (9) and (10)) were used (Atchison and Brown, 1957):

$$\sigma_{\ln \text{UEV}} = \sqrt{\ln \omega_{\text{UEV}}} \quad (9)$$

$$\mu_{\ln \text{UEV}} = \ln(\text{UEV}) - 0.5(\sigma_{\ln \text{UEV}}) \quad (10)$$

The resulting set of UEV approximations (100) provide a distribution from which the left and right sides of the confidence interval can be estimated by the 2.5 and 97.5 percentile values, respectively. In order to get a representative sample, this procedure was executed 100 times thus generating 100 distributions (for a total of 10 000 UEV values). From each distribution, the mean, median, and standard deviation values were reported, and these values were averaged across the 100 distributions to arrive at average values for each UEV. From the average mean and standard deviation, the  $\sigma_{\text{geo}}^2$  value for that UEV was estimated according to Eq. (1).

**Table 4**  
UEV uncertainty estimated from the analytical solution.

Item	UEV den.	UEV (sej/Den.)	Parameter $\mu_{geo}$	Parameter $\sigma_{geo}^2$	Model and/or scenario $\sigma_{geo}^2$ <sup>a</sup>	Total $\mu_{geo}$	Total $\sigma_{geo}^2$	Lower UEV using parameter uncertainty	Upper UEV using parameter uncertainty	Lower UEV using total uncertainty	Upper UEV using total uncertainty
Lead	g	5.46E+12	4.85E+12	2.59	9.12	2.57E+12	11.09	1.87E+12	1.26E+13	4.38E+11	5.38E+13
Iron	g	1.06E+10	1.15E+10	2.00	6.66	7.18E+09	7.53	5.73E+09	2.29E+10	1.52E+09	8.63E+10
Petroleum	J	1.21E+05	9.78E+04	3.59	1.04	9.77E+04	3.59	2.72E+04	3.51E+05	2.72E+04	3.51E+05
Groundwater	g	9.36E+05	8.90E+05	1.86	1.28	8.83E+05	1.95	4.78E+05	1.66E+06	4.56E+05	1.74E+06
Labor	J	6.74E+06	6.73E+06	1.08	11.43	3.11E+06	11.44	6.26E+06	7.24E+06	5.89E+05	7.70E+07

<sup>a</sup> All values represent model uncertainty, except for labor for which this is scenario uncertainty.

The stochastic simulation did not incorporate the model and scenario uncertainty components, which could only be estimated by way of the analytical solution. The stochastic simulation recalculates the UEV by varying the parameters, but does not incorporate uncertainty from use of alternative models or on account of parameters from other scenarios. Thus to compare the stochastic and analytically-derived results from parameter uncertainty, the calculated *parameter*  $\sigma_{geo}^2$  (Eq. (5)) may be compared with the  $\sigma_{geo}^2$  value obtained from the simulation distributions.

Uncertainty was also estimated for two UEVs calculated with the table-form model—electricity from oil and sulfuric acid made from secondary sulfur. The emergy tables used to estimate these two UEVs were simplified to include only items that contributed in total to 99% of the emergy in these items. Uncertainty was estimated solely with the Monte Carlo simulation routine used for the formula UEVs, with the following change: uncertainty data in the form of  $\sigma_{geo}^2$  values for both inventory values (e.g. secondary sulfur in g in Table 5) and their respective UEVs (e.g. UEV for secondary sulfur in sej/g) were used in conjunction with their means to create random lognormal variables for use in the simulation. Estimation of the natural log-form of the standard deviation for these variables for generating lognormal random values was slightly different than for the formula UEV case, because it used the  $\sigma_{geo}^2$  value instead of the sample standard deviation (Eq. (11)).

$$\sigma_{\ln UEV} = \frac{\ln \sigma_{geo}^2}{1.96} \quad (11)$$

The uncertainty factors in the Ecoinvent Unit Processes library for geometric variance were used for the  $\sigma_{geo}^2$  values for the inventory data (Ecoinvent Centre, 2007). For the UEVs of the inventory items, the deterministic mean and the geometric variance of the UEV for the same item calculated with the formula model were used when appropriate as the mean and  $\sigma_{geo}^2$  value, respectively. This choice was based on the assumption that the inventory items (e.g. water to make sulfuric acid) had the same UEV as those calculated with formula UEV models (e.g. groundwater).

The 95% confidence interval of the simulation distributions for formula and the table-form UEVs were compared with the confidence intervals predicted by a perfect log-normal distribution ( $\mu_{geo}(x \div) \sigma_{geo}^2$ ), those predicted by a normal-lognormal hybrid distribution using the arithmetic mean as the center parameter ( $\mu(x \div) \sigma_{geo}^2$ ), and those predicted by a normal distribution ( $\mu \pm 1.96\sigma$ ). Eqs. (1)–(3) were used to estimate the  $\mu_{geo}$  and  $\sigma_{geo}^2$  from the  $\mu$  and  $\sigma$  derived from the sample distribution. The percent difference between the predicted and model distribution tails was calculated to measure the how accurately the predicted distributions represented the model distribution.

### 3. Results

The details of the uncertainty calculations for lead are shown in Table 3. For lead, parameter and model uncertainty were estimated. The  $\sigma_{geo}^2$  values (approximately the upper tail of the distribution divided by the median) for the five parameters range from 1.03 to 2.25. The total parameter uncertainty ( $\sigma_{geo}^2$ ) is larger than the largest individual parameter  $\sigma_{geo}^2$  value, but less than the sum of these parameter  $\sigma_{geo}^2$  values. The total uncertainty for lead, consisting of the combined model and parameter uncertainty (without scenario uncertainty) is dominated by the model uncertainty, which has a large  $\sigma_{geo}^2$  value due to large differences in previously published estimates used for the UEV of lead. The 95% confidence interval for the lead UEV using this analytical form of estimation would vary across three orders of magnitude, from 4.38E+11–5.38E+13 sej/g. However it the UEV model used to estimate the mean was the only acceptable model, the interval would

**Table 5**  
Emergy summary with uncertainty of 1 kg of sulfuric acid.<sup>a</sup>

No	Item	Data (units)	Unit	Relative data uncertainty $\sigma_{geo}^2$	UEV (sej/unit)	Relative UEV uncertainty $\sigma_{geo}^2$	Solar emergy (sej)
1	Secondary sulfur	2.14E+02	g	1.32	5.20E+09	3.59	1.11E+12
2	Diesel	3.41E+03	J	1.34	1.21E+05	3.59	4.13E+08
3	Electricity	6.30E+04	J	1.34	3.71E+05	2.77	2.34E+10
4	Water	2.41E+05	J	1.23	1.90E+05	1.95	4.57E+10
5	Product Sulfuric acid	1.00E+03	g		1.18E+09	3.31	1.18E+12

<sup>b</sup>CI<sub>95</sub> = 8.10E+08 (x/÷) 3.31

Notes: 1. Secondary sulfur is a by-product of oil-refining, and is assigned the UEV of petroleum = (UEV of petroleum, sej/J)/(enthalpy of oil, J/g) 2. UEV of petroleum from this paper. 3. From this paper. 4. (UEV for groundwater, sej/g)/(4.94 J/g).

<sup>a</sup> Inventory data from Ecoinvent 2.0 (Ecoinvent Centre, 2007).

<sup>b</sup> Example of incorporation of a confidence interval into an emergy table assuming a lognormal distribution.

**Table 6**  
UEV Monte Carlo results and comparison of model CI's with lognormal, hybrid, and normal confidence intervals.<sup>a</sup>

Item	UEV type <sup>b</sup>	Monte Carlo results		Model 95% CI		Predicted 95% CIs					
		$\mu_{geo}$	$\sigma_{geo}^2$	Lower	Upper	Lognormal CI		Hybrid CI		Normal CI	
						Lower error	Upper error	Lower error	Upper error	Lower error	Upper error
Lead	F	5.19E+12	2.73	1.93E+12	1.38E+13	-1.5%	2.6%	12%	17%	-123%	-11%
Iron	F	1.30E+10	1.99	6.62E+09	2.53E+10	-1.8%	2.3%	4.5%	8.8%	-40%	-6.6%
Petroleum	F	1.57E+05	3.55	4.66E+04	5.44E+05	-4.5%	2.9%	18%	27%	-273%	-14%
Ground H <sub>2</sub> O	F	9.40E+05	1.92	5.06E+05	1.77E+06	-2.9%	2.4%	2.6%	8.3%	-35%	-5.8%
Labor	F	6.91E+06	1.08	6.45E+06	7.40E+06	-0.32%	0.35%	-0.25%	0.42%	-0.57%	0.12%
Electricity from oil	T	2.81E+05	2.77	1.16E+05	7.68E+05	-12%	2.4%	0.85%	17.3%	-126%	-11%
Sulfuric acid	T	8.10E+08	3.31	2.72E+08	2.67E+09	-10%	0.50%	31%	47%	-179%	-96%

<sup>a</sup> Confidence intervals defined as follows: Lognormal =  $\mu_{geo} (x \div) k$ ; hybrid =  $\mu (x \div) k$ ; normal =  $\mu \pm 1.96\sigma$ .

<sup>b</sup> F = formula UEV; T = table-form UEV. UEVs are in sej/g for lead, iron, groundwater, and sulfuric acid, and sej/J for petroleum, labor, and electricity from oil.

shrink to 1.87E+12 – 1.26E+13 sej/g, indicating considerably less uncertainty.

The geometric variance calculations from the analytical solution for the formula UEVs (lead, iron, petroleum, groundwater, and labor) showed a wide range of values presented in Table 4. Geometric variance values were dominated by model or scenario variances in the cases of the minerals and labor. The total parameter uncertainty ranged from 1.08 for labor to 3.59 for petroleum, whereas model uncertainty was as high as 9.12 for lead.

The confidence intervals estimated from the analytical and stochastic methods were of similar breadth (for all five formula UEVs), although they were not identical – the intervals from the analytical solution were all shifted slightly to the left.

The Monte Carlo simulation of the UEVs produced largely right-skewed distributions, as indicated by the means for UEVs (see column 3 of Table 4 being greater than the medians).

Without exception the means of the simulated UEV distributions were less than the medians.

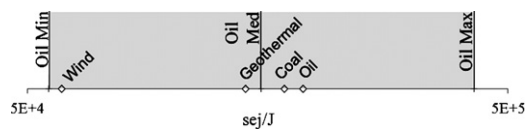
The table-form UEV calculation for sulfuric acid appears in Table 5. The geometric variance values for the inputs of secondary sulfur and diesel are those calculated for petroleum<sup>5</sup>; the UEV for diesel is that calculated for petroleum; the UEV for electricity from oil was calculated from an emergy table and the geometric variance is the  $\sigma_{geo}^2$  value from the Monte Carlo simulation; and the UEV and geometric variance for water are those calculated above for groundwater. The Monte Carlo simulation resulted in a median of 8.10E+8 and a  $\sigma_{geo}^2$  value of 3.31. The other table-form UEV, electricity, also had a  $\sigma_{geo}^2$  value less than that of its major input, petroleum, suggesting a pattern of less breadth in the confidence intervals of table-form UEVs than those of their most variable input.

<sup>5</sup> Assuming the geometric variance is the same because they share similar UEV models, which is an assumption mentioned later in the discussion.

Table 6 summarizes the results of the Monte Carlo simulations for all UEVs when the parameter distributions were assumed lognormal, and compares the resulting confidence intervals against those that would be predicted by lognormal, hybrid, and normal distributions. A number of notable differences are present between these results and those of the calculated uncertainty values for formula UEVs in Table 6. The UEV means from the simulation are higher in all cases than the deterministic means presented in Table 4, but the simulation median values are lower than the deterministic means. The  $\sigma_{geo}^2$  values from the simulation, which were calculated according to Eq. (1) from the average mean and standard deviations of the Monte Carlo distributions, are not identical to the parameter geometric variance values from Table 4; however, the Monte Carlo  $\sigma_{geo}^2$  values were always  $\pm 5\%$  of the analytically calculated geometric variances.

The lognormal confidence interval was the best fit for the simulated UEV distributions: error of the lognormal approximation of either the lower or upper tail was never larger than 5%, except for the lower tail of the two table-form UEVs. However this distribution tended to consistently overestimate the confidence interval.<sup>6</sup> The hybrid distribution tended to predict a distribution shifted to the right of the model with increased error, and the normal distribution often predicted a lower tail many orders of magnitude less than the model value. The smaller the standard deviation relative to the mean (reflected by the  $\sigma_{geo}^2$  value), the better all predicted distributions fit the model interval. In the case of the two table-form UEVs, electricity from oil and sulfuric acid, the lognormal confidence interval tended to underpredict the model lower tail more severely (suggesting that the tail is closer to the mean), but was still the best fit when considering the combined error in both tails. The left tail of these model UEV distributions was more con-

<sup>6</sup> This could be in part be explained by the fact that the Eq. (3) is more precisely for a 95.5% confidence, rather than a 95.0%, confidence interval (Limpert et al., 2001).



**Fig. 2.** Published UEVs for electricity by source (diamonds on axis) from Brown and Ulgiati (2002), superimposed upon a modeled range of the petroleum UEV, using the geometric variance for electricity from oil ( $\sigma_{\text{geo}}^2 = 2.77$ ) calculated in this paper.

stricted, and in these cases the quotient of the model mean and  $\sigma_{\text{geo}}^2$  value, reflected by the hybrid model, was a closer approximate of the lower tail.

#### 4. Discussion and conclusions

To fully characterize uncertainty for UEVs, the sources of uncertainty need to be identified and quantified. The classification scheme introduced by the EPA provides a useful framework which helps in identification of quantifiable aspects of uncertainty. However in practice, describing the uncertainty in parameters, scenarios and models requires significant effort and must draw from previous applications of various models and across various scenarios. In this manuscript, the data sufficient to characterize these three types of uncertainty for each UEV was not readily available, and as a result in no cases has a total parameter uncertainty been estimated that includes all parameter, model, and scenario uncertainty for lack of either multiple models or modeled scenarios from which to include that component of uncertainty. Unless one or more of these types of uncertainty can be categorically determined to be absent for a UEV, the uncertainty measures presented here underestimate the total uncertainty in these UEVs.

Acknowledging this underestimate, how much uncertainty are in unit energy values? Parameters for describing the uncertainty ranges inherit in 7 UEVs have been presented and analyzed here. Informally, emergy practitioners may have assumed an implicit error range of “an order of magnitude”, but this analysis reveals such a general rule of thumb is inappropriate. As quantified here the UEVs may vary with either less or more than one order of magnitude, but this is UEV specific. However, when UEVs have as their basis the same underlying models, if the parameters specific to one or more of UEVs have a similar spread, then the UEV uncertainty should be similar. Thus, as was demonstrated here, uncertainty values for a UEV may be co-opted from an UEV calculated with the same model (e.g. minerals in the ground) with reasonable confidence if original estimation is infeasible. Adoption of geometric variances from UEVs calculated with the same model would provide an advantage as a reasonable estimation of uncertainty rather than a vague or undefined measure.

Quantifying model uncertainty may have implications regarding the certainty of comparative evaluations. Fig. 2 shows the UEVs estimated for different types of electricity in Brown and Ulgiati (2002)—all fall within the range of confidence interval of the UEV for petroleum, estimated from the mean UEV reported by the authors and the geometric variance calculated for this electricity type in this paper (2.77), using Eqs. (5) and (6) to estimate the median and Eq. (3) to estimate the tails. Although it appears that from this analysis the UEVs of electricity sources would be statistically similar, this ignores the fact that many of the same UEVs are used in the inputs to these electricity processes. Hypothetically, if the same UEVs are used as inputs to processes being compared, relative comparisons can still be made, all of the variance due to the UEVs of inputs is covariance. This represents a problem of applying this uncertainty model to rank UEVs where there is strong covariance, which is not addressed here.

#### 4.1. Comparing the analytical and stochastic solutions

Multiple advantages of proceeding with an analytical solution have been listed in the risk analysis literature. These include the ability to partition uncertainty among its contributing factors and identify factors contributing to the greatest uncertainty in a model (Rai and Krewski, 1998) as well as the greater simplicity of calculation (Slob, 1994). Further advantages suggested here in the context of UEVs are the ability to include other sources of uncertainty which cannot be quantified in a simple Monte Carlo analysis, and the ability to replicate the values for geometric variance.

However, because table-form UEVs are the most common form of emergy evaluation, and the stochastic simulation method is the only method presented which is functional for this form of unit emergy calculations, the stochastic method is likely to be more useful to emergy practitioners.

Model and scenario uncertainty components, which were not quantified in the Monte Carlo simulation, can be particularly significant in emergy, due to the fact that emergy values for a product are often used across a wide breadth of scenarios, computed with alternative models, and adopted in subsequent evaluations by other authors without knowledge of the context in which the original UEVs were calculated. The most desirable solution to these problems with uncertainty would be: first for model uncertainty, to agree on the use of consistent models for a UEV type to eliminate the discrepancy that occurs between competing models; for scenario uncertainty, to make UEVs more scenario specific whenever possible to eliminate scenario uncertainty. Where elimination of this model and scenario uncertainty is not possible, an alternative would be to develop a more complex version of stochastic model that would include estimation of model and scenario uncertainty in addition to parameter uncertainty.

Following from what is predicted mathematically, this study confirmed that formula UEVs as multiplicative products fit a lognormal distribution better than a normal distribution. Table-form UEVs, while they are sumproducts, also tended to be better described by lognormal distributions than normal distributions, although the two UEVs simulated both fits this distribution to a lesser degree than the formula UEVs. Using the deterministic mean as the center parameter for a multiplicative confidence interval, represented by the hybrid approach, may be a tendency of emergy practitioners for simplified description of confidence intervals, but was shown here to result in more error than using the median, except for the estimate of the lower tail of the confidence interval for table-form UEVs.

#### 4.2. Conclusions

Ultimately the accuracy of UEV uncertainty measures depends upon the representativeness of the statistics describing the model parameters. In this case a broad but not exhaustive attempt was made to describe uncertainty and variability in the model factors for the UEVs evaluated. For this reason, this author recommends sources of uncertainty be further investigated and more thoroughly quantified before they are propagated for use in future studies. The responsibility should rest with authors to diligently seek out and to summarize the uncertainty in parameters they adopt, and to perpetuate that uncertainty with the UEV uncertainty both to present the uncertainty of their own work and so that it can be adopted by those that use this UEV in the future.

By describing uncertainty associated with emergy estimates, emergy is more likely to become adopted as a measure of cumulative resource use or for other purpose in LCA. Description of uncertainty in parameters and across models and scenarios will increase transparency in emergy calculations, thus answering one of the critiques which has hindered wider adoption (Hau and

Bakshi, 2004). Uncertainty descriptors, namely the geometric variance, can be used along with inventory uncertainty data to calculate uncertainty in estimates of total emergy in complex life cycles. It can be further be used to compare different life cycle scenarios with greater statistical confidence. Pairing UEVs with uncertainty data and identifying sources of uncertainty will also help emergy practitioners understand and report the statistical confidence of their calculated emergy values and to prioritize reduction of uncertainty as a means to improve the accuracy of emergy values.

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### Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at [doi:10.1016/j.ecolmodel.2009.10.032](https://doi.org/10.1016/j.ecolmodel.2009.10.032).

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