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Emergy algebra: Improving matrix methods for calculating transformities

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ABSTRACT

Transformity is one of the core concepts in Energy Systems Theory and it is fundamental to the calculation of emergy. Accurate evaluation of transformities and other emergy per unit values is essential for the broad acceptance, application and further development of emergy methods. Since the rules for the calculation of emergy are different from those for energy, particular calculation methods and models have been developed for use in the emergy analysis of networks, but double counting errors still occur because of errors in applying these rules when estimating the emergies of feedbacks and co-products. In this paper, configurations of network energy flows were classified into seven types based on commonly occurring combinations of feedbacks, splits, and co-products. A method of structuring the network equations for each type using the rules of emergy algebra, which we called "preconditioning" prior to calculating transformities, was developed to avoid double counting errors in determining the emergy basis for energy flows in the network. The results obtained from previous approaches, the Track Summing Method, the Minimum Eigenvalue Model and the Linear Optimization Model, were reviewed in detail by evaluating a hypothetical system, which included several types of interactions and two inputs. A Matrix Model was introduced to simplify the calculation of transformities and it was also tested using the same hypothetical system. In addition, the Matrix Model was applied to two real case studies, which previously had been analyzed using the existing method and models. Comparison of the three case studies showed that if the preconditioning step to structure the equations was missing, double counting would lead to large errors in the transformity estimates, up to 275 percent for complex flows with feedback and co-product interactions. After preconditioning, the same results were obtained from all methods and models. The Matrix Model reduces the complexity of the Track Summing Method for the analysis of complex systems, and offers a more direct and understandable link between the network diagram and the matrix algebra, compared with the Minimum Eigenvalue Model or the Linear Optimization Model.

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1. Introduction

The development of Energy Systems Theory (Odum, 1971, 1983, 1994) has given rise to the recognition that the maximum power principle (Lotka, 1922a,b) is a 4th law of thermodynamics that describes the behavior of non-equilibrium systems (Odum, 1996). The operation of the 4th law leads to the ubiquitous positive feedback loops and hierarchical structures seen in nature (Odum, 1994), because these designs increase power in their networks (Odum, 1996; Campbell, 2001). As a result, Odum (1996) recognized that energy flows of the universe are organized into an energy transformation hierarchy, which he proposed as a possible 5th law of thermodynamics or perhaps a corollary to the 4th law. Hierarchical organization, which means that many units at one level contribute to fewer units at a higher level, with the higher level, in turn, controlling those at the lower level (Allen and Starr, 1982; Brown et al., 2004; Allen et al., 2008), has been confirmed by many scientific studies as a common property of environmental, ecological, and economic systems (Odum, 1996; Huang, 1998; Lan et al., 2002). The flow of available energy through hierarchical structures gives rise to two fundamental quantities, emergy and transformity. In this context, it is the flow of emergy (empower) that is maximized by the operation of the 4th law and transformity gives the position of any energy flow within a hierarchical structure (Odum, 1996). Since emergy flow is equal to a flow of available energy times its transformity, the practical application of emergy analysis requires information on both available energy and transformity. Whereas, information on the energy content of environmental and economic flows is widely available, the transformities of these quantities are often unknown or estimated with first order approximations

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(Campbell, 2003; Campbell et al., 2005). As a result accurate information on the transformities of quantities and flows of a system under study is often the limiting factor in performing an emergy evaluation.

The importance of accurate and reproducible determinations of transformities has been recognized by many scientists performing emergy evaluations (Odum, 2000; Odum et al., 2000; Brown and Bardi, 2001; Brandt-Williams, 2002; Kangas, 2002; Cohen et al., 2007; Lu and Campbell, 2009). Odum (1996) gave a list of 10 methods for determining an unknown transformity and several authors including Collins and Odum (2000), Bardi et al. (2005) have explored matrix methods for determining the transformities of energy flows in an ecosystem network. The accurate calculation of transformities and other emergy per unit values is fundamental for both the further development of theories related to emergy and the application of emergy methods. In addition, accurate calculation of transformities could also lead to further insights and developments in hierarchy theory by providing the basis for predicting the location and mode of action of components and processes within hierarchical network structures. Therefore, this study focuses on further development of the matrix methods to calculate transformities.

1.1. Emergy and transformity

Since this paper is concerned with the estimation of transformities as a necessary factor to determine changes in the emergy flow (empower) in various systems, it behooves us to consider these two concepts in more detail. Emergy is the available energy of one kind previously used up directly and indirectly to make a product or service (Odum, 1996). The unit of emergy is the emjoule (Scienceman, 1987), where the prefix “em-” connotes the past use of energy that was required to create the present product or service. Transformity is the emergy of one kind, e.g., solar joules, required to make a unit of available energy of a product or service. The solar joule is commonly used as the base unit for emergy studies. In this case solar transformities are expressed as solar emjoules per joule (sej/J).

Transformity is a core concept of Energy Systems Theory (Odum, 1994), because it is fundamental to the calculation of emergy and it is an indicator of the relative position of components and flows within an energy systems network. Transformities (sej/J) and other emergy per unit values, e.g., specific emergy (sej/g), are often the most difficult pieces of information to find or estimate in order to complete an emergy evaluation. Transformities, specific emergies, and other emergy per unit values, if available, could be used to unify all kinds of knowledge on energy, material, information and money flows, which now cannot be compared directly (Odum, 1996; Lan et al., 2002; Brown and Ulgiati, 2004; Cavalett et al., 2006; Martin et al., 2006). For example, accurate transformities of various pollutants would allow trading schemes to be carried out fairly across disparate quantities, e.g., nitrogen, carbon, acid, heat, by converting them to a common basis as emergy measured in solar emjoules (Odum, 1996).

1.2. The relationship between emergy and transformity

The fundamental equation of emergy analysis can be derived from the definition for transformity given above:

$$\text{emergy (sej)} = \text{transformity (sej/J)} \times \text{available energy (J)} \quad (1)$$

This equation is in linear form so that at steady state a set of emergy flows in a network can be calculated from a set of coefficients (the transformities) times a set of available energy flows, if the network interactions are specified and the transformities of the external emergy sources are known. However, difficulties in the matrix calculations may arise because while the calculation of emergy is based on the underlying flows of available energy, which conform to the conservation principle; in all cases, emergy itself is not conserved. This nonconformity to the conservation law arises because emergy is defined in terms of what was directly and indirectly required for, or used up in, the production process of an item. Thus, emergy is a second law quantity in the sense that its value depends on the available energy used up in a production process. As a result, emergy is not conserved across a production process when co-products are produced, because the total emergy input to the process is required for each co-product. Therefore, emergy has a unique set of rules that govern its calculation and are formulated to deal with problems that arise from accounting for co-products and feedback in a network (Tennenbaum, 1988; Odum, 1996; Bastianoni and Marchettini, 2000; Lan et al., 2002; Giannantoni, 2003). In fact the calculation methods used to determine the emergy delivered by feedback and contained in co-products are the most important differences between emergy analysis and energy analysis (Brown and Herendeen, 1996).

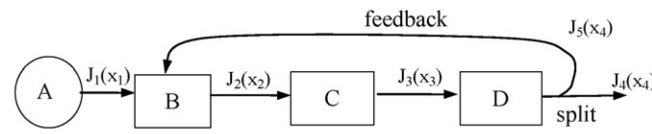

Fig. 1. Single feedback (Type 1).

1.3. Past studies and research questions

The method and models that have been used to determine the emergy of a pathway are as follows: (1) The Track Summing Method (Tennenbaum, 1988; Odum, 1996). In this method transformity is determined by tracing energy flows back from the product through the energy systems network to each initial input emergy. However, when the ecosystem network under study is complex, this calculation process can be complicated (Campbell, 2004). (2) Patterson (1983) constructed a matrix equation to express the energy transformation processes of a system by assuming the conservation of a quantity like emergy for every single product across a production process operating at steady state. Quantities similar to transformities or specific emergies would be obtained by solving the matrix equation with the linear algebra method. (3) Further simplifying this model, Collins and Odum (2000) provided a program using *Mathematica* to solve the matrix equation with a Minimum Eigenvalue Model. Later, Thompson et al. (2007) introduced an approach to solve the same model in *Microsoft Excel*. (4) Bardi et al. (2005) employed a Linear Optimization Model based on the Newton Iterative Search to estimate the matrix equation with the help of the Solver tool in *Microsoft Excel*. However, the examination of similar case studies using both models along with the Track Summing Method showed that there were double counting errors remaining after solving for the transformities with these models (Collins and Odum, 2000; Odum and Collins, 2003; Bardi et al., 2005), because they departed from the rules for emergy algebra when handling feedbacks and co-products. The following questions are of particular interest in this paper: “How did these errors occur and can they be corrected?” and “Is there a credible method of solving for transformities compared with the existing methods?”

2. Methods

2.1. Overview of methods

In this paper, the possible combinations of feedback and co-product pathways that can occur in an energy systems network were classified into seven common types (Figs. 1–7). Then, the rules

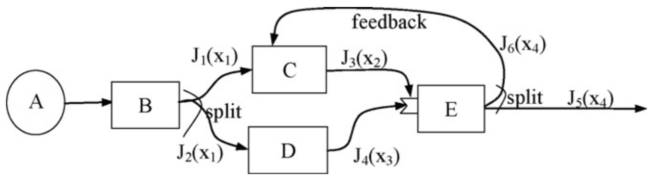

Fig. 2. Combination of a split with feedback (Type 2).

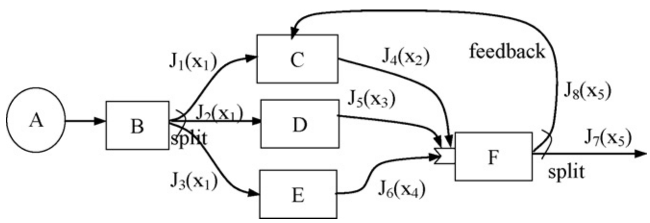

Fig. 3. Combination of a multiple split with feedback (Type 3).

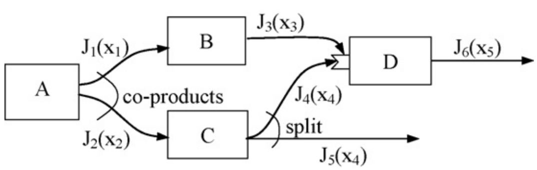

Fig. 4. A co-product is split and then recombined (Type 4).

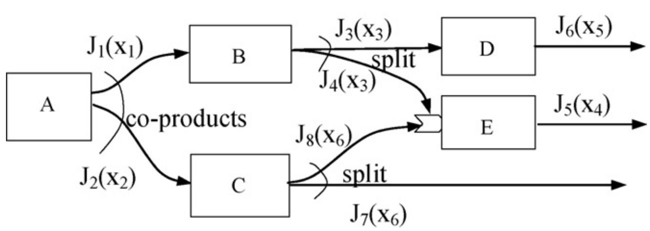

Fig. 5. Both co-products are split and then recombined (Type 5).

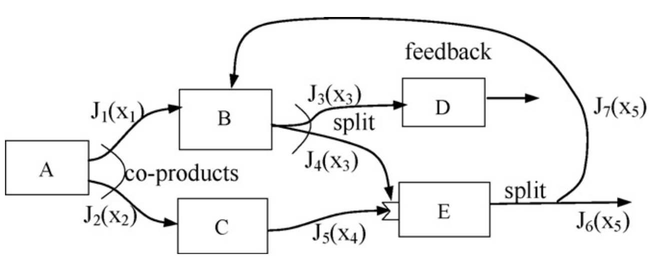

Fig. 6. A co-product is split and then recombined with feedback (Type 6).

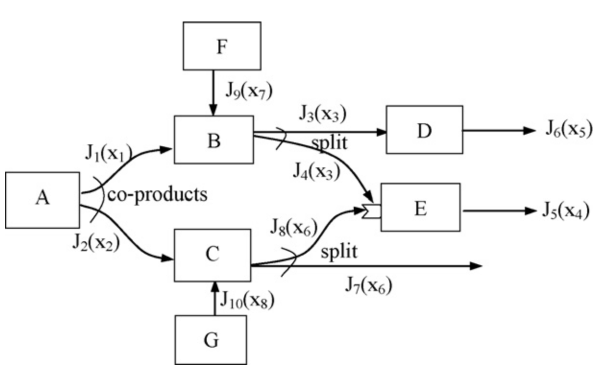

Fig. 7. Co-products are split after additional emergy sources enter the network (Type 7).

of emergy algebra were applied, systematically, to simplify each type of interaction often occurring in a system network, a process that we called preconditioning. Next, a hypothetical energy systems network with feedback, split and co-product flows was constructed and used to test the methods for handling complex flows.

The preconditioning process applied to this network consisted of inspecting the network diagram and identifying the presence of one or more of the seven configuration types defined below (Figs. 1–7). After the existing method and models were used to calculate the transformities of the hypothetical system, we defined a Matrix Model and used it to evaluate the hypothetical network for comparison with the results obtained from the existing method and models. The Matrix Model was structured to further simplify the calculation process and to build a more direct link between the emergy evaluation of the network and the mathematical process used. The reliability of the Matrix Model was checked by applying it to two case studies, which had been calculated using the existing method and models.

2.2. *The rules of emergy algebra*

The rules for handling co-products and feedback pathways were first referred to as emergy algebra by Scienceman (1987). A systematic statement of these rules was given in chapter 6 of Odum (1996), and a comparison of the calculation rules for embodied energy and emergy was demonstrated by Brown and Herendeen (1996). The rules of emergy algebra are as follows:

(1) For a system at steady state, all the emergy inflows to a production process are assigned to the outputs.
(2) When an output pathway splits into two or more pathways of the same type, the emergy input is assigned to each 'leg' of the split based on its fraction of the total energy or material flow on the pathway; therefore, the transformity or specific emergy of each branch of the split is the same.
(3) For a process with more than one output, i.e., co-products, each output pathway from the process carries the total emergy input to the process, i.e., the entire emergy required for a process is also required for each of the products.
(4) No emergy input to a system can be counted twice. Thus, if an input or feedback flow to a component is derived from itself, i.e., it carries emergy already counted in the emergy required for the component, then the input or feedback flow is not added to the emergy required for the component, i.e., input emergy is not double-counted. A corollary to the prohibition against double counting is that co-products of the same production process when reunited cannot be added to obtain an emergy input greater than the original emergy input. Thus, when adding emergy inflows or outflows that are co-products, only the largest one should be considered.

2.3. *Methods to simplify networks by classifying interactions*

An interactive network of energy flows contains many feedbacks, which gives rise to the possibility of many variations of the double counting problem. Double counting problems appear when feedback, split and co-product flows interact in a network either through the recombination of co-products or from feedbacks. To lower the risk of miscalculating the emergy required for a pathway, we classified the potential interactions into seven common types (Figs. 1–7). Three of these types (Figs. 1–3) are related to handling feedbacks either alone or from splits and four (Figs. 4–7) are related to the recombination of feedback and splits derived from co-products. Of course, there are many possible combinations of these processes found in the networks encountered over the range of organizational scales that occur in environmental systems. However, we believe that these seven types represent the most common configurations where double counting might arise in performing an emergy analysis. We hypothesized that if these configuration types were used to structure the system of linear equations prior to emergy analysis using the methods of linear algebra, double counting errors would be avoided and a simpler and more understandable relationship between the network of emergy flows and the mathematics would be obtained, i.e., the emergy balance equations would be constructed according to the rules of emergy algebra. We have termed this process of structuring the equations representing the energy systems network by applying configuration types, preconditioning.

2.3.1. Single feedback and feedbacks from split flows

A single chain of components has one emergy source A (see Fig. 1, where J represents the energy flow each pathway carries and x represents the transformity of the pathway, similar terminology is used in Figs. 2–7). The feedback flow from D into B should not be added to the input emergy of B to avoid double counting the input from A. Thus, the emergy balance equations at steady state are

$$\begin{cases} B: J_1x_1 = J_2x_2 \\ C: J_2x_2 = J_3x_3 \\ D: J_3x_3 = (J_4+J_5)x_4 \end{cases} \tag{2}$$

Emergy splits into two pathways that recombine as a result of feedback from a component higher in the network (Fig. 2). To avoid double counting, only the part of the emergy in the feedback flow from D through E to C should be counted in the emergy required for C, i.e. $(J_6/(J_5+J_6))J_4x_3$; the emergy input to E came from C and D, i.e., $J_1x_1+J_4x_3$, but not $J_3x_2+J_4x_3$, because J_3x_2 contains the feedback emergy from D, i.e., $J_3x_2=J_1x_1+(J_6/(J_5+J_6))J_4x_3$, and the overlap between J_3x_2 and J_4x_3 is $(J_6/(J_5+J_6))J_4x_3$. Therefore, the constructed emergy balance equations at steady state are

$$\begin{cases} D: J_2x_1 = J_4x_3 \\ C: J_1x_1 + \dfrac{J_6}{J_5+J_6}J_4x_3 = J_3x_2 \\ E: J_1x_1 + J_4x_3 = (J_5+J_6)x_4 \end{cases} \tag{3}$$

For the case when an energy or material inflow splits into n branches, the calculation is more complicated, but it can be handled similarly. For the case where n equals 3 (Fig. 3), the emergy of the feedback flow from F to C that should be counted to avoid double counting the input to C on J_1x_1 is only the part derived from D and E, i.e. $(J_8/(J_7+J_8))(J_5x_3+J_6x_4)$.

2.3.2. Feedback and splits from co-product energy flows

The emergy of co-products is recombined within the web of interactions as the result of a split of one co-product (Fig. 4). The total emergy flow into D is only that on J_3x_3, which is derived from B. This emergy inflow is larger than the emergy arriving from the other co-product, J_4x_4, through C, because it has undergone a split. To avoid double counting, only the larger emergy from B or C, i.e., J_3x_3, should be counted as the emergy input to D. Hence, the emergy balance equations at steady state are

$$\begin{cases} A: J_1x_1 = J_2x_2 \\ B: J_1x_1 = J_3x_3 \\ C: J_2x_2 = (J_4+J_5)x_4 \\ D: J_3x_3 = J_6x_5 \end{cases} \tag{4}$$

The emergy of co-products is recombined as a result of splits of the two co-products (Fig. 5). Since the emergy inflows from B and C to E are derived from the co-products of A, only the larger of these two flows should be counted. The larger flow is denoted by $\max\{J_4x_3, J_8x_6\}$ which defines the emergy input to E, i.e., $J_5x_4=\max\{J_4x_3, J_8x_6\}$. Therefore, the emergy balance equations at steady state are

$$\begin{cases} A: J_1x_1 = J_2x_2 \\ B: J_1x_1 = (J_3+J_4)x_3 \\ C: J_2x_2 = (J_7+J_8)x_6 \\ D: J_3x_3 = J_6x_5 \\ E: J_5x_4 = \max\{J_4x_3, J_8x_6\} \end{cases} \tag{5}$$

The emergy of co-products is recombined in the web with feedback to one of the co-product branches (Fig. 6). The total emergy flow into E is J_5x_4, because due to the split of the output from B, the emergy flow from B to E is less than that from C to E. Also, the total emergy flow into B is only J_1x_1, because the emergy of the feedbacks from B through E to B and from C through E to B come from the co-products of A and their inclusion would double count this original input. Therefore, the constructed emergy balance equations at steady state are

$$\begin{cases} A: J_1x_1 = J_2x_2 \\ B: J_1x_1 = (J_3+J_4)x_3 \\ C: J_2x_2 = J_5x_4 \\ E: J_5x_4 = (J_6+J_7)x_5 \end{cases} \tag{6}$$

Please notice the difference between the handling of feedback from splits and co-products by comparing with Type 2 above (Fig. 2).

The same rules apply to more complicated configurations in which co-products are split after additional emergy sources enter the system. The emergy input to component E in Fig. 7 is determined as follows: part of the emergy inflow from B to E and from C to E comes from the co-products of A, and only the larger one of these two should be added to the emergy input to E. In addition, the split fraction of the emergy entering the network from independent sources F and G must be added to the larger input from the co-products, i.e. $(J_4/(J_3+J_4))J_9x_7+(J_8/(J_7+J_8)J_{10}x_8)+\max\{(J_4/(J_3+J_4))J_1x_1, (J_8/(J_7+J_8)J_2x_2)\}$, to determine the total emergy input to E. Thus, the emergy balance equations at steady state are

$$\begin{cases} A: J_1x_1 = J_2x_2 \\ B: J_1x_1 + J_9x_7 = (J_3+J_4)x_3 \\ C: J_2x_2 + J_{10}x_8 = (J_7+J_8)x_6 \\ D: J_3x_3 = J_6x_5 \\ E: J_5x_4 = \dfrac{J_4}{J_3+J_4}J_9x_7 + \dfrac{J_8}{J_7+J_8}J_{10}x_8 \\ \quad + \max\{\dfrac{J_4}{J_3+J_4}J_1x_1, \dfrac{J_8}{J_7+J_8}J_2x_2\} \end{cases} \tag{7}$$

3. Results and discussion

3.1. A hypothetical case study for demonstrating the application of the preconditioning method

A hypothetical network of energy flows supported by two emergy sources (Fig. 8) was constructed for the purpose of testing the preconditioning process designed to avoid double counting errors and comparing the solutions obtained from network analysis after preconditioning to the results obtained from the existing method and models for determining transformities. The hypothetical network (Fig. 8) contained various kinds of interactions among feedbacks, splits and co-products and it represented most of the configuration types identified in Figs. 1–7.

Upon inspection of the network shown in Fig. 8, the following configurations can be observed:

(1) Splits: Split flows divide the emergy in proportion to the energy flow on the branches and thus each flow has the same transformity, e.g., x_2, x_5, x_6, x_7 and x_{10} in Fig. 8. Although split flows are independent of each other, double counting may result from their recombination with feedback or co-products, e.g., the recombination of the flows from D and E as inputs to component G, or the feedback from component E to H, in Fig. 8.

(2) Co-products. The network in Fig. 8 has one co-product configuration. D and E have the same emergy input from the co-product flows leaving B, i.e., $300x_3 = 400x_4$, but when they converge again into G, only the larger one is counted. The emergy of both co-products splits as it passes through D and E on the way to G, and since $(50/250) \times 300x_3 = (50/250) \times 400x_4 < (200/300) \times 400x_4$, the emergy from E, i.e., $200x_7$, is taken as the input to G, instead of the emergy from D.

(3) Feedbacks: The pattern of feedbacks in Fig. 8 shows that the emergy flow from S_2 through H to B and the feedback from A that passes through C, E and H to B are independent of the input from A to B; therefore, the emergy from S_2 and the feedback from A through C and E should be added to the input emergy to B along with the direct input from A. Thus, the total emergy input to B is $500x_2 + (100/300) \times 500x_{11} + (100/300) \times (100/300) \times 200x_5$. Similarly, the total emergy input to E is $500x_2 + (100/300) \times 500x_{11} + 200x_5$, instead of $400x_4 + 200x_5$, because the co-product flow into E, $400x_4$, which is equal to the total emergy input to B, contains emergy feedback from E through H and B equal to $(200/9)x_5$, which overlaps with the emergy $200x_5$ entering E from C. Furthermore, the total emergy input to H is $500 \times x_{11} + (100/300) \times 500x_2 + (100/300) \times 200x_5$, instead of $500 \times x_{11} + 100 \times x_7$, because $100x_7$ contains part of the emergy from S_2 that has traveled through H and B, i.e. $(100/300) \times (100/300) \times 500x_{11}$.

To avoid double counting and easily estimate the emergy input at any point in a complex network of flows, we propose that the independent sources be tracked and counted in turn to determine the emergy input to any component or process as described in the Track Summing Method (Tennenbaum, 1988; Odum, 1996).

3.2. Application of preconditioning to the existing method and models

The Track Summing Method, the Minimum Eigenvalue Model and the Linear Optimization Model were tested and reviewed in detail through their application to the hypothetical network shown in Fig. 8.

3.2.1. The Track Summing Method

According to the 1st rule of emergy algebra, the emergy input to a process is equal to the emergy of each output at steady state. After preconditioning, we can address each process in the network of energy flows from left to right by dividing the total emergy input to a process by the corresponding energy output to obtain the transformity of the process. This method was called the Track Summing Method first developed by Tennenbaum (1988) and it was presented in Odum (1996).

Following preconditioning, i.e., the application of the types of feedback, split, and co-product recombinations discussed in Section 3.1 to structure the set of linear equations, the transformities (x_i)

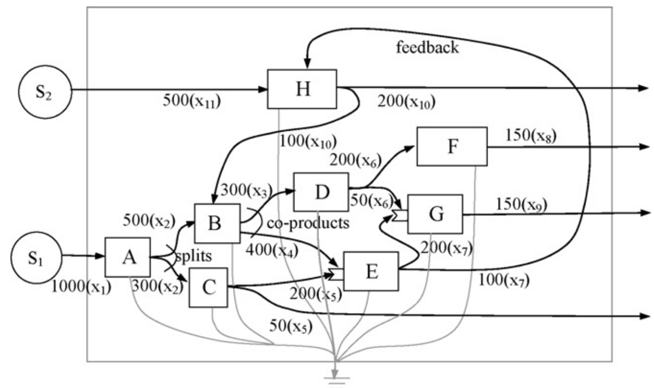

Fig. 8. An evaluated hypothetical energy systems network with various combinations of interactions among feedback, split and co-products flows (unit: J/time).

of the energy flows (numbers on pathways) in the system shown in Fig. 8 were calculated using the following formulae (8).

Assume the transformity of the known source S_1 in Fig. 8 is k times that of S_2, i.e., $x_1 - k \times x_{11} = 0$, and let $k = 1$, $x_1 = 1$ sej/J, i.e., $x_1 = x_{11} = 1$ sej/J, here for simplicity:

$$\begin{cases} A: 1000x_1 = 800x_2 \\ B: 500x_2 + \dfrac{100}{300} \times 500x_{11} + \dfrac{100}{300} \times \dfrac{100}{300} \times 200x_5 = 300x_3 \\ B: 500x_2 + \dfrac{100}{300} \times 500x_{11} + \dfrac{100}{300} \times \dfrac{100}{300} \times 200x_5 = 400x_4 \\ C: 300x_2 = 250x_5 \\ D: 300x_3 = 250x_6 \\ E: 500x_2 + \dfrac{100}{300} \times 500x_{11} + 200x_5 = 300x_7 \\ F: 200x_6 = 150x_8 \\ G: 200x_7 = 150x_9 \\ H: 500x_{11} + \dfrac{100}{300} \times 500x_2 + \dfrac{100}{300} \times 200x_5 = 300x_{10} \\ x_1 = 1; \quad x_{11} = 1 \end{cases} \tag{8}$$

And then, the transformity vector that resulted was

$$X = [\,1 \quad 1.25 \quad 2.75 \quad 2.06 \quad 1.5 \quad 3.3 \quad 3.64 \quad 4.4 \quad 4.85 \quad 2.69 \quad 1\,]^T$$

3.2.2. The Minimum Eigenvalue Model

Patterson (1983) constructed a series of linear equations as a description of the energy transformation processes according to the principle that quality adjusted energy (emergy) is conserved across the production process of every single unit at steady state, and then used the coefficient matrix to express the energy systems network, i.e., each row showed a transformation process equation and each column expressed a type of energy. As a result, all of the transformities could be obtained at one time by solving the matrix equation with linear algebra methods. Based on this work, Collins and Odum (2000) provided a program in *Mathematica* to solve the matrix equation with the Minimum Eigenvalue Model.

To illustrate the application of the Minimum Eigenvalue Model, we used the system shown in Fig. 8 and applied the simplifying configurations from Section 3.1 to structure the equations for analysis. The result of preconditioning the linear equation group was shown in Eq. (9) below, where terms were listed in the order that

Table 1
The coefficient matrix M.

To	From										
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x_2	1000	−800	0	0	0	0	0	0	0	0	0
x_3	0	500	−300	0	200/9	0	0	0	0	0	500/3
x_4	0	500	0	−400	200/9	0	0	0	0	0	500/3
x_5	0	300	0	0	−250	0	0	0	0	0	0
x_6	0	0	300	0	0	−250	0	0	0	0	0
x_7	0	500	0	0	200	0	−300	0	0	0	500/3
x_8	0	0	0	0	0	200	0	−150	0	0	0
x_9	0	0	0	0	0	0	200	0	−150	0	0
x_{10}	0	500/3	0	0	200/3	0	0	0	0	−300	500
x_{11}	1	0	0	0	0	0	0	0	0	0	−1

they appeared in the matrix:

$$\begin{cases} A: 1000x_1 - 800x_2 = 0 \\ B: 500x_2 - 300x_3 + \dfrac{200}{9}x_5 + \dfrac{500}{3}x_{11} = 0 \\ B: 500x_2 - 400x_4 + \dfrac{200}{9}x_5 + \dfrac{500}{3}x_{11} = 0 \\ C: 300x_2 - 250x_5 = 0 \\ D: 300x_3 - 250x_6 = 0 \\ E: 500x_2 + 200x_5 - 300x_7 + \dfrac{500}{3}x_{11} = 0 \\ F: 200x_6 - 150x_8 = 0 \\ G: 200x_7 - 150x_9 = 0 \\ H: \dfrac{500}{3}x_2 + \dfrac{200}{3}x_5 - 300x_{10} + 500x_{11} = 0 \\ x_1 - x_{11} = 0 \end{cases} \tag{9}$$

The equation group was transformed into matrix form as $M \times x = 0$ (Table 1).

The matrix M in the Minimum Eigenvalue Model has n unknown variables, but there are only $n-1$ independent equations. Therefore, this linear equation group has an infinite number of solutions. But the equations $(M^T \times M) \times X = 0$ and $M \times X = 0$ have the same solutions, which can be proved using matrix algebra. The matrix $(M^T \times M)$is an nth-order square matrix with a rank of $n-1$, and because its rank is less than its order, it is a singular matrix and its determinant is equal to 0. This means that it has no inverse, thus it must have an eigenvalue of 0, i.e., $L^\theta = 0$. Thus, there exists a non-zero eigenvector X^θ corresponding to the eigenvalue L^θ, i.e. $(M^T \times M) \times X^\theta = L^\theta \times X^\theta = 0$ and then $M \times X^\theta = 0$, and this eigenvector is the vector of transformities when the absolute value of its elements is normalized by the minimum value. Based on this principle, we ran the program developed by Collins and Odum (2000) to calculate the transformities of the evaluated network using the Minimum Eigenvalue Model (For details, see Appendix B online). The transformity vector obtained was

$$X = [\,1 \quad 1.25 \quad 2.75 \quad 2.06 \quad 1.5 \quad 3.3 \quad 3.64 \quad 4.4 \quad 4.85 \quad 2.69 \quad 1\,]^T$$

3.2.3. The Linear Optimization Model

Bardi et al. (2005) employed a Linear Optimization Model called the Newton Iterative Search to solve the matrix equation $M \times X$=0, which can be realized through application of the Solver tool in *Microsoft Excel*.

In a manner similar to that used for the Track Summing Method and the Minimum Eigenvalue Model, we employed the evaluated network in Fig. 8 to illustrate the application of preconditioning to the Linear Optimization Model. After preconditioning the structure of the set of linear equations using the configuration classes in Section 3.1, we calculated the transformities of the evaluated network using the Linear Optimization Model (For details, see Bardi et al. (2005) or Appendix B online). The transformity vector obtained was

$$X = [\,1 \quad 1.25 \quad 2.75 \quad 2.06 \quad 1.5 \quad 3.3 \quad 3.64 \quad 4.4 \quad 4.85 \quad 2.69 \quad 1\,]^T$$

3.2.4. Consideration of the pros and cons of the existing method and models

As a result of the assumption that steady state conditions exist in a network of processes and the concomitant condition that emergy is conserved across the production process for an individual product, transformities can be calculated using any of the methods considered above. For the same energy systems diagram, we have shown that the Track Summing Method (Tennenbaum, 1988; Odum, 1996), the Minimum Eigenvalue Model (Collins and Odum, 2000; Odum and Collins, 2003), and the Linear Optimization Model (Bardi et al., 2005) all produced the same vector of transformities, after the structure of the linear equations was prepared by preconditioning.

Although there is a simple theory behind the Track Summing Method, in practice the step-by-step visual tracking processes used for calculating emergy inputs can be complicated, especially for large networks. The Minimum Eigenvalue Model can automatically calculate all of the transformities at one time, but the specific mathematical processes are relatively complex and the linkage between the rules of emergy algebra and the mathematical process is not straightforward. The Linear Optimization Model employs the Newton Iterative Search through a set of constraint conditions to calculate all of the transformities using the Solver tool in *Microsoft Excel*. Considering the ease of access to the software, this model may have wide application, although the linkage between the matrix algebra and the energy systems diagram is not straightforward. Thompson et al. (2007) also introduced a method to solve the Minimum Eigenvalue model for transformities using the tools available in *Microsoft Excel*.

3.3. The Matrix Model

During the review of the existing method and models, we found that, if we brought the external inputs into the initial conditions, as a member of the equation group to be solved by the Minimum Eigenvalue Model, an equation group with n unknown variables and n independent equations was obtained. This equation group can be transformed into a $n \times n$ square matrix equation, which has a unique solution instead of an infinite number of solutions. This allowed us to easily obtain all the transformities directly by solving the matrix equation using the standard method of matrix inversion.

To compare this model with the other method and models, we analyzed the system in Fig. 8 once again. The linear equation group was constructed in Eq. (10), after preconditioning the equations by applying corrections for the types of configurations examined above:

$$\begin{cases} x_1 = 1 \\ A: 1000x_1 - 800x_2 = 0 \\ B: 500x_2 - 300x_3 + \dfrac{200}{9}x_5 + \dfrac{500}{3}x_{11} = 0 \\ B: 500x_2 - 400x_4 + \dfrac{200}{9}x_5 + \dfrac{500}{3}x_{11} = 0 \\ C: 300x_2 - 250x_5 = 0 \\ D: 300x_3 - 250x_6 = 0 \\ E: 500x_2 + 200x_5 - 300x_7 + \dfrac{500}{3}x_{11} = 0 \\ F: 200x_6 - 150x_8 = 0 \\ G: 200x_7 - 150x_9 = 0 \\ H: \dfrac{500}{3}x_2 + \dfrac{200}{3}x_5 - 300x_{10} + 500x_{11} = 0 \\ x_{11} = 1 \end{cases} \tag{10}$$

The equation group was transformed into matrix form as $A \times X = F$, where A is an energy flow matrix, X is a transformity vector and F is the corresponding constant vector:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1000 & -800 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 500 & -300 & 0 & \frac{200}{9} & 0 & 0 & 0 & 0 & 0 & \frac{500}{3} \\ 0 & 500 & 0 & -400 & \frac{200}{9} & 0 & 0 & 0 & 0 & 0 & \frac{500}{3} \\ 0 & 300 & 0 & 0 & -250 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 300 & 0 & 0 & -250 & 0 & 0 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 & 200 & 0 & -300 & 0 & 0 & 0 & \frac{500}{3} \\ 0 & 0 & 0 & 0 & 0 & 200 & 0 & -150 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 200 & 0 & -150 & 0 & 0 \\ 0 & \frac{500}{3} & 0 & 0 & \frac{200}{3} & 0 & 0 & 0 & 0 & -300 & 500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{11}$$

After the initial source condition, $x_1 = 1$ in this case, was brought into the matrix equation of input-output flows, the matrix equation, $A \times X = F$, included n unknown variables and n independent equations. Such a matrix equation has only one solution. Therefore, the transformity vector X could be easily obtained by solving $X = A^{-1} \times F$ directly in, *MatLab*. Also, this calculation can be performed using the matrix functions, mmult and minverse, in *Microsoft Excel*, as shown in Appendix B. The transformity vector obtained from both these methods was

$$X = [\,1 \quad 1.25 \quad 2.75 \quad 2.06 \quad 1.5 \quad 3.3 \quad 3.64 \quad 4.4 \quad 4.85 \quad 2.69 \quad 1\,]^T$$

For the sake of convenience, an input-output flow matrix can be constructed directly from the energy systems network diagram under the constraint that the sum of the emergy inputs to each component or unit is equal to the emergy of each output flow. The rules for constructing the input-output flow matrix for an energy systems network are shown in Eq. (12):

$$(Unit_i \to Unit_j) = a_{ji} = \begin{cases} 1, & \text{when } i = j \text{ and unit}_i \text{ is a source.} \\ -J_{ij}, & \text{when } i = j\text{('}-\text{' means energy output.)} \\ J_{ij}, & \text{when } i \neq j\text{('}+\text{' means energy input.)} \\ 0, & \text{when no energy flows into unit}_j. \end{cases} \tag{12}$$

where $Unit_i \to Unit_j$ means energy flows from $Unit_i$ to $Unit_j$, defined as a_{ji} ($A = (a_{ji})n \times n$, where n is the number of all units), and J_{ij} is its corresponding energy value.

The corresponding element of vector F in $A \times X = F$ is its transformity when $Unit_i$ is an initial known source, or the element will be defined to be 0, for example $F = [\,1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1\,]^T$, in the above matrix Eq. (11).

We examined the importance of preconditioning the structure of the network equations by constructing an input-output flow matrix A with both external inputs and internal components, once again using the evaluated network in Fig. 8. These equations were structured solely based on the assumption that the emergy inflow will equal the emergy of the output across any production process, i.e., co-product, split and feedback rules were applied, but adjustments for the various combinations and configurations of these processes as applied in "preconditioning" were not made. The input-output flow matrix A (Eq. (13)) was obtained directly from the diagram in Fig. 8, by applying the rules given in Eq. (12):

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1000 & -800 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 500 & -300 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 500 & 0 & -400 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 300 & 0 & 0 & -250 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 300 & 0 & 0 & -250 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 400 & 200 & 0 & -300 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 200 & 0 & -150 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 50 & 200 & 0 & -150 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 & -300 & 500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

Because preconditioning was not performed for Eq. (13) when determining the emergies for flows in the complex systems network (Fig. 8), we hypothesized that double counting errors would be prone to occur, because all the rules of emergy algebra were not applied to the equation structure. The transformity vector calculated for this matrix was

$$X = [\,1 \quad 1.25 \quad 3.09 \quad 2.32 \quad 1.5 \quad 3.71 \quad 4.09 \quad 4.95 \quad 6.70 \quad 3.03 \quad 1\,]^T$$

Since the energy systems network in Fig. 8 has coupled feedback, split and co-product flows, matrix A (Eq. (13)) must be modified to avoid double counting by using a preconditioning step to adjust for the configurations identified in Section 3.1.The preconditioned equations are shown in Eq. (14). In addition, to avoid miscalculating the emergy of flows, when preconditioning the equations for complex flow configurations such as the feedback flow into B (Fig. 8), it may be advisable to first write out the equation: $500x_2 + (100/300) \times 500x_{11} + (100/300) \times (100/300) \times 200x_5 - 300x_3 = 0$, based on the rules of emergy algebra and then bring the coefficients into the appropriate row of matrix A, in this case, row 3 as shown in Eq. (14). In other simpler cases, the original input-output flow matrix can be modified directly:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1000 & -800 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 500 & -300 & 0 & \frac{200}{9} & 0 & 0 & 0 & 0 & 0 & \frac{500}{3} \\ 0 & 500 & 0 & -400 & \frac{200}{9} & 0 & 0 & 0 & 0 & 0 & \frac{500}{3} \\ 0 & 300 & 0 & 0 & -250 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 300 & 0 & 0 & -250 & 0 & 0 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 & 200 & 0 & -300 & 0 & 0 & 0 & \frac{500}{3} \\ 0 & 0 & 0 & 0 & 0 & 200 & 0 & -150 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 200 & 0 & -150 & 0 & 0 \\ 0 & \frac{500}{3} & 0 & 0 & \frac{200}{3} & 0 & 0 & 0 & 0 & -300 & 500 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

After preconditioning, the transformity vector obtained was $X = [\,1 \quad 1.25 \quad 2.75 \quad 2.06 \quad 1.5 \quad 3.3 \quad 3.64 \quad 4.4 \quad 4.85 \quad 2.69 \quad 1\,]^T$. Compared to the result without preconditioning, i.e., $X = [\,1 \quad 1.25 \quad 3.09 \quad 2.32 \quad 1.5 \quad 3.71 \quad 4.09 \quad 4.95 \quad 6.70 \quad 3.03 \quad 1\,]^T$, seven of the 11 transformities calculated were different, with the difference ranging from 12 to 38 percent. This result clearly shows the importance of preconditioning for accurately calculating the emergies and transformities in complex flow networks. This transformity vector was the same as that obtained using the Track Summing Method, the Minimum Eigenvalue Model and the Linear Optimization Model after preconditioning the equations to take into account the complex flow relationships that might result in double counting. The emergy flow diagram for Fig. 8 is shown in Fig. 9.

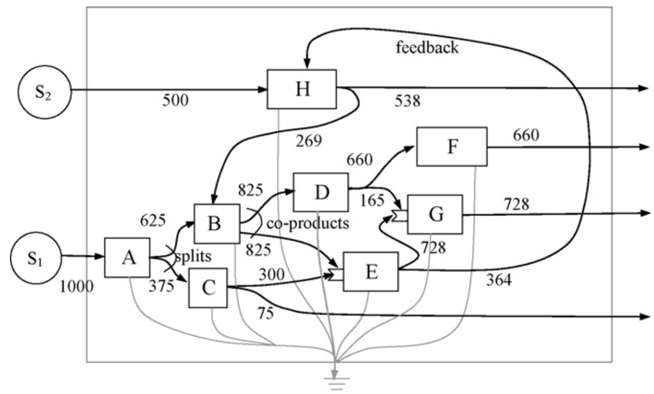

Fig. 9. The emergy flow network diagram of Fig. 8 (unit: sej/time).

The Matrix Model allowed us to construct an input-output energy flow matrix equation directly from the energy systems diagram. Once this was accomplished applying the preconditioning steps given above, the transformities of the flows can be obtained by simple matrix inversion. Thus, the Matrix Model not only offers a simpler calculation of transformities compared with the Track Summing Method, but also is easier to understand and perform than the Minimum Eigenvalue and Linear Optimization Models.

3.4. Case studies

The Silver Springs Ecosystem (Collins and Odum, 2000) and a Louisiana Oyster Reef System (Odum and Collins, 2003; Bardi et

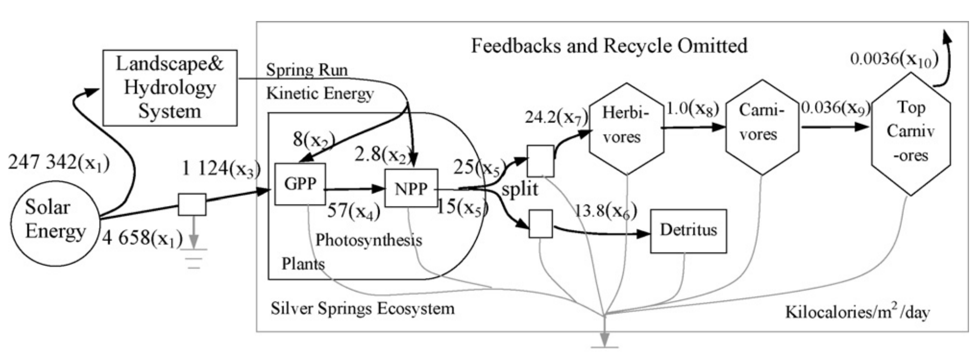

Fig. 10. Silver Springs Ecosystem Model (Collins and Odum, 2000).

Table 2
Comparison of transformities for the Silver Springs, Florida ecosystem calculated using three methods.

Transformity	Track Summing Method[a]	Minimum Eigenvalue Model[a]	Matrix Model	Error percent (%)
x_1 Solar energy	1	1	1	0
x_2 Kinetic energy	22 902	22 902	22 902	0
x_3 Light into plants	4.1	4.14413	4.14	0[c]
x_4 GPP[b]	4 385	3296.04	3296.04	33.04
x_5 NPP[b]	6 300	6 300	6 300	0
x_6 Organics to detritus	6 630	6 847.83	6 847.83	3.18
x_7 Organics to herbivores	6 529	6 508.26	6 508.26	0.32
x_8 Herbivores to carnivores	158 000	157 500	157 500	0.32
x_9 Carnivores to top carnivores	4.4E+06	4.375E+06	4.375E+06	0[c]
x_{10} Top carnivores	4.4E+07	4.375E+07	4.375E+07	0[c]

[a] Collins and Odum (2000).
[b] GPP: gross primary production; NPP: net primary production.
[c] The differences among the results of three methods only came from their precision instead of the error from models.

al., 2005) were analyzed to test the effectiveness and reliability of the Matrix Model compared to results obtained from applying the existing method and models to the same systems. Since small discrepancies existed between the estimates of transformities obtained using the different methods of calculation, we hoped that the preconditioning process along with the more straightforward Matrix Model would help us identify errors, if they existed.

The following input-output flow matrix equation for the Silver Springs Ecosystem (Fig. 10) can be directly constructed as shown in Eq. (15) below using the rules in Eq. (12):

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 247342 & -10.8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4658 & 0 & -1124 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 1124 & -57 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.8 & 0 & 57 & -40 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15 & -13.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 25 & 0 & -24.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 24.2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -0.036 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.036 & -0.0036 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

And then the transformity vector calculated using the Matrix Model is

$$X = [1, 22\,902,\ 4.14,\ 3296.04,\ 6300,\ 6847.83,\ 6508.26,\ 157\,500,\ 4\,375\,000, 43\,750\,000]^T$$

This result is the same as that obtained from the Minimum Eigenvalue Model, but not for 4 of the transformities calculated through the Track Summing Method used in Collins and Odum (2000), with the error raging from 0.3 to 33 percent (Table 2). The transformities differ, because the emergy flows into GPP and NPP were calculated incorrectly in their application of the Track Summing Method. For example, the emergy flow into GPP should be $8x_2 + 1124x_3 = 187\,872$, instead of 252 000 or 247 342, because only 8 J in 10.8 J of the kinetic energy flowed into GPP (see Fig. 5 in Collins and Odum (2000)). If the emergy flows into GPP and NPP were modified by a preconditioning process, the result of the Matrix Model would agree with that of the Track Summing Method.

The input-output flow matrix for the Oyster Reef Ecosystem is constructed below using the methods and assumptions given above

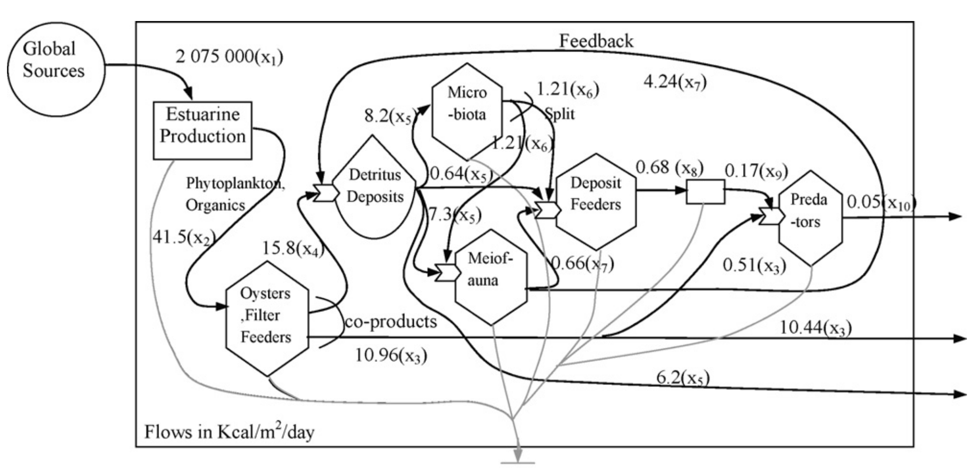

Fig. 11. Oyster Reef Ecosystem Model (Bardi et al., 2005).

Table 3
Comparison of transformities for the Oyster Reef System calculated using three different models.

Transformity[a]	Minimum Eigenvalue Model[b]	Linear Optimization Model[c]	Matrix Model	Error percent (%)
x_1 Solar energy	1.00E+00	1.00E+00	1.00E+00	0
x_2 Phytoplankon/organic	5.00E+04	5.00E+04	5.00E+04	0
x_3 Oyster to predators	1.89E+05	1.89E+05	1.89E+05	0
x_4 Oyster to detritus	–	–	1.31E+05	–
x_5 Detritus deposit	3.46E+05	3.46E+05	9.29E+04	272
x_6 Microbiota yield	1.18E+06	1.18E+06	3.15E+05	275
x_7 Meiofauna yield	8.07E+05	8.07E+05	2.16E+05	274
x_8 Deposit Feeder yield	3.19E+06	3.19E+06	8.57E+05	272
x_9 See Fig. 11	–	–	3.43E+06	–
x_{10} Predator yield	1.28E+07	1.28E+07	1.17E+07	9

[a] Each x_i of three models should correspond to its latter object and then the transformity sequences, of Odum and Collins (2003) and Bardi et al. (2005) were respectively adjusted in the columns 2 and 3 according to the energy systems diagram in Fig. 11. And "–" means x_4 and x_9 were not calculated by the Minimum Eigenvalue Model and the Linear Optimization Model.
[b] Odum and Collins (2003).
[c] Bardi et al. (2005).

in Eq. (12):

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2075000 & -41.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 41.5 & -10.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 41.5 & 0 & -15.8 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 15.8 & -22.34 & 0 & 4.24 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 8.2 & -2.42 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7.3 & 1.21 & -4.9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.64 & 1.21 & 0.66 & -0.68 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & -0.17 & 0 \\
0 & 0 & 0.51 & 0 & 0 & 0 & 0 & 0 & 0.17 & -0.05
\end{bmatrix}$$

Co-product flows are coupled to feedback flows in this system, so the input-output flow matrix should be modified by preconditioning the matrix of equations. The following configurations were identified and reflected in the structure of the equations:

(1) Flows with transformities x_3 and x_4 are co-products and when these co-products converge and flow into "Predators" (Fig. 11), only the larger emergy flow $0.17 \times x_9$ should be counted to avoid double counting the emergy from "Oysters, Filter Feeders".
(2) The emergy flow into "Detritus Deposits" from "Meiofauna" is a feedback flow derived from Detritus Deposits, and so it should not be added again to the total emergy flow required for detritus.

After preconditioning the structure of the equations the matrix equation for the Oyster Reef System is

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2075000 & -41.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 41.5 & -10.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 41.5 & 0 & -15.8 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 15.8 & -22.34 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 8.2 & -2.42 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 7.3 & 1.21 & -4.9 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.64 & 1.21 & 0.66 & -0.68 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.68 & -0.17 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.17 & -0.05
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix}
=
\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\qquad (16)$$

And the transformity vector calculated will be

$$X = [\,1, \quad 50\,000, \quad 189\,325, \quad 131\,329, \quad 92\,883, \quad 314\,727, \quad 216\,094, \quad 857\,186, \quad 3\,428\,744, \quad 11\,657\,729\,]^T$$

This solution is different from the results of Odum and Collins (2003) and Bardi et al. (2005), with the error raging from 9 to 275 percent (Table 3). This difference is entirely due to the failure to assiduously follow the rules of emergy algebra in structuring the model equations for calculating the emergy of flows. For example, the emergy flow into Predators that was derived from co-products, x_3 and x_4 was counted twice. In addition, the feedback emergy flow from "Meiofauna" into Detritus Deposits was double-counted. Such errors may be understandable, because of the difficulty of accurately tracing emergy through a complex network; however, in all cases these discrepancies were removed by applying the preconditioning methods presented in this paper to structure the matrix equations before solving for transformities. When the equation structures of the models analyzed in the other studies were modified using preconditioning to take care of the double counting configurations, the results were the same as those obtained with the Matrix Model.

4. Conclusion

Given a network of energy flows based on a set of external emergy inputs, the transformities of the energy flows within the network can be determined by the Track Summing Method, the Minimum Eigenvalue Model, the Linear Optimization Model, and the Matrix Model. All four approaches are based on the conservation of emergy across the production process for a single unit at steady state and give valid results, but they use different mathematical methods to find the solution. The Track Summing Method calculates the emergy input to each process in the network proceeding from left to right and divides the total emergy input to a component by the corresponding energy output to obtain the transformity. Although the calculation mechanism is simple, the entire process may be complicated for complex systems. The Minimum Eigenvalue Model constructs a matrix equation, $M \times X = 0$, and then solves for the eigenvector that corresponds to the minimum absolute value of the eigenvalues of the matrix $M^T \times M$ in $(M^T \times M) \times X = 0$ to obtain the transformities. Although the Minimum Eigenvalue Model can simultaneously estimate all of the transformities at one time using a computer, the specific processes are relatively complex and difficult to understand. The Linear Optimization Model employs the Newton Iterative Search through the constraint conditions to solve for all the transformities using the Solver tool in *Microsoft Excel*. This approach benefits from the wide availability and ease of use associated with *Excel*; therefore, it may have a greater potential for use, compared with the Minimum Eigenvalue Model. However, Thompson et al. (2007) gave a method for solving the Minimum Eigenvalue Model in *Excel*, which lessens the advantage of using the Linear Optimization Method in *Solver*. The Matrix Model allows the direct construction of an input-output matrix equation, $A \times X = F$, to express the energy systems network. Furthermore, the matrix equation has only one solution, i.e., $X = A^{-1} \times F$, and it can be easily solved in both *MatLab* and *Excel*.

No matter which method is employed, it is essential that preconditioning be used to structure the matrix equations in a manner that accounts for configurations of feedbacks, splits, and co-products that can result in large errors in the transformities due to double counting, if not handled properly. After preconditioning the equation structures of the networks examined in this study, the results of all the methods and models tested were the same. When many co-products and splits arise from storages and flows and feedback to each other, it could be difficult to precondition the network equations by inspection. Although given a network diagram, it is possible to perform such complex preconditioning using the inspection method; an automated computer model to perform the preconditioning process will be desirable, especially for complex networks.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ecolmodel.2009.10.015.

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