COMBINING ENERGY LAWS AND COROLLARIES OF THE MAXIMUM POWER PRINCIPLE WITH VISUAL SYSTEMS MATHEMATICS

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This conference was convened to consider opportunities for mathematics to relate to problems of ecology and man and, vice versa, to search in the problems of man and nature for new opportunities for mathematics development. What are appropriate mathematics, languages, and conceptual theory for the large scale system of man and nature as unified by intertwining energy flows, material cycles, and regulatory relationships of order and disorder?

All real events are accompanied by flows of energy irrespective of the realm of size. All causal actions require energy transformation. Mathematics and models to have reality must include the constraints and characteristics of systems that are generated as a consequence of energy laws. If mathematics can be wedded to the energy laws and models developed according to the way energy flow generates order and process, general energy models can be developed that may be a basic point of departure for models of any size dimension. Wild options of creative mathematical thinking are thus severely restricted by energy constraints.

Macroscopic Scale of Size

At every level of organization, energy flows according to its nature, developing structure and pattern appropriate to its size scale. For example, the macroscopic system of man and nature develops cities, highways, mosaics of

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ecosystem types, power networks, information flow webs, and drainage basins of the water cycle. Many features are best seen in aerial photography whereas others are felt more intimately as one goes through a day of existence as part of the system. A man is rarely able to see his macroscale system well since he is within it. Since energy operates man's world with the same restrictions and roles as it operates other size realms, man has evolved in a role of adapting to the energy requirements of him. By this view the complex activities of man are means for fine tuning his programmatic role in fitting himself and his landscapes to the energy requirements for survival.

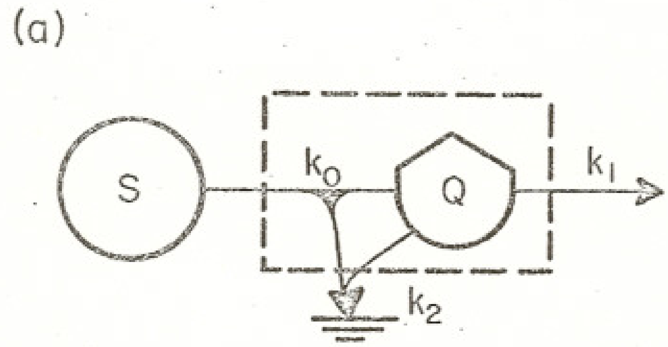
Visual Mathematics

Special opportunity exists for union of ecology and mathematics in further development of comprehensive visual diagrammatic mathematics. Many are already developing diagrammatic systems procedures, since this is a natural means for representing real world processes. There may be a national trend at this. Each of these symbolic systems has some special aspects to offer, and it may be possible to combine these diagrams as a synthesis to represent many concepts of science, mathematics, and engineering in visual presentations. At this conference, for example, Forrester's diagrams, Levins' loop analysis, and our energy diagrams were used among others. Configurations have the same information as equations or matrices but to many are more easily thought about.

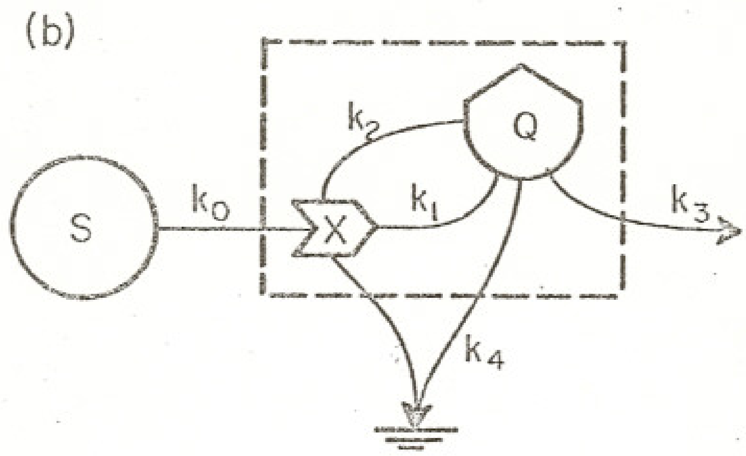
We need to combine the best features of all diagrammatic systems languages. Synthesis of diagrams provides an opportunity to include energy constraints. If energy flows generate order and process, energy diagrams are a natural means of generalizing models of man and nature so as to include natural aggregations and processes. While translating and combining concepts, visual mathematics may be easier for thinking, faster for teaching, and understood by more people. Energy budgets provide convenient overall summaries of the resources of a model and help the modeller recognize what is important according to its energy impact. In this essay pictorial circuit language is used to discuss the energy laws that control real systems and thus should constrain models. Energy circuit language is used to help retain the deterministic constraints of energy with survival criteria and to bring the energy know-how of other size levels of study into effective use for the environmental scale. Gradually we hope to incorporate more and more of other visual mathematical ideas and symbols into a unified energy circuit language, presently called "energese" for short.

Energy Circuit Language

In Fig. 1 diagrams show how basic patterns of order are built by energy flow operating with three energy principles and their corollaries. Energy circuit language is used continuing our effort to generalize. In Fig. 2 are some of the energese symbols with their kinetic and energetic translations and just a few of the notations, conventions, and rules for the language. A verbal introduction was given in book form (Odum, [2]; papers that deal with physical, chemical, and mathematical basis of these symbols are as follows [3,4]. Numerical values of flows, transfer coefficients, algebra, etc. may be added as needed.

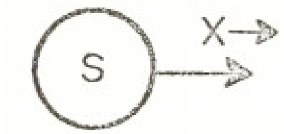


$$\dot{Q} = k_0 S - k_1 Q - k_2 Q$$



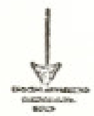
$$\dot{Q} = (k_1 - k_2)SQ - k_3 Q - k_4 Q$$

Fig. 1. Diagram illustrating Law of Conservation of Energy.
(a) Storage and linear pathways; (b) autocatalytic configuration which we call pseudo-logistic.

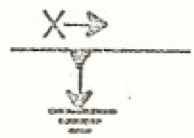


$$X = f(s)$$

- External energy source (s) delivers forcing function (x) to a pathway.

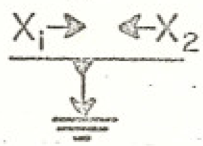


- Heat sink indicates entropy increase with energy dispersal to environment.



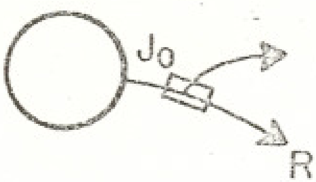
$$J = \frac{1}{R} X$$

- Normal pathway with flux (J) driven by force (x) opposed by frictional, dissipative backforce.



$$J = \frac{1}{R}(X_1 - X_2)$$

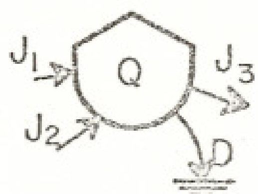
- Same but with backforce from downstream energy storage (x_2).



$$J = kR$$

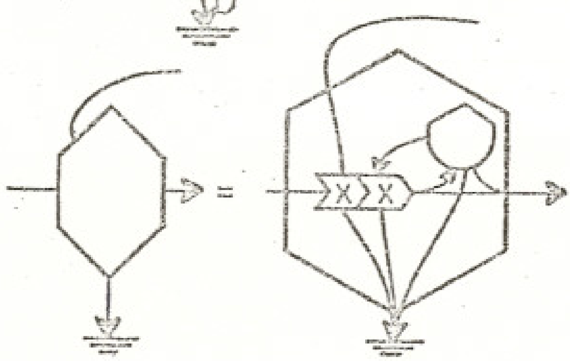
$$J = \frac{kJ_0}{1+k}$$

- Source limited external source delivers limited flow (J_o) which is tapped by flux dependent pathway (J).



$$\dot{Q} = J_1 + J_2 - J_3 - D$$

- Storage symbol for state variables drawn to resemble a tank.



- Self maintaining units with upgraded storage, autocatalytic pumping, and various related self organizing pathways are indicated with hexagon class symbol.

Fig. 2. Some basic configurations of energy circuit language.

Visual Representation of Energy Laws

Illustrated by Fig. 1 is the first energy law: All energies pass from the external source (circles) into storages (tank symbols) leaving the system as energy export (outbound arrows crossing system boundaries). All energy inflowing must be accounted for in internal storage reservoirs (tank symbol) or in flows to the outside.

Also illustrated by Fig. 1 is the provision that the second energy law be shown. All energy flowing in a process must have part of its energy degraded in concentration so as to increase the entropy of the environment mainly by dispersion of heat. This loss is shown with the heat sink symbol that indicates where heat or other energy concentrations are degraded and exported. All storage reservoirs must continually lose energy into degraded form also because of the second energy principle. Hence every process has a heat sink symbol (See Fig.2) which helps one keep track of energy flow through processes and losses from storages. Depreciation of storages and the energy degradation inherent in real process are shown leaving the system together at the bottom of diagrams in Fig. 1.

Maximum Power Principle and Corollaries

A third energy principle used to develop visual mathematics is less well known. Apparently first stated by Lotka in 1924 [1], the maximum power principle combines natural selection with energetics and general systems thinking. In brief, the maximum power principle indicates:

Survival of Systems That Maximize Their Power Utilization

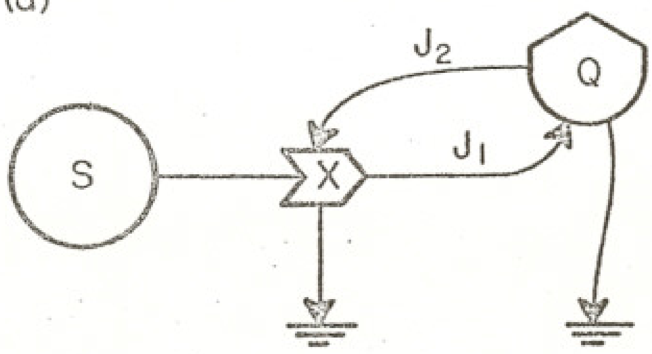
The principle is probably self convincing in the same way that natural selection of species by "survival of the fittest" is intuitively correct for most thinkers. Proof of complete generality of principles is probably not possible and like the second energy principle stands because of the absence of contradictions. Systems that have more energy can do more to predominate, meet contingencies, survive stress, and build means for the short run and the long run. Maximizing power may be the design criterion for the universe; it provides a reference against which other propositions may be compared.

The implications of the maximum power selection are best made by discussing the aspects of the models in Fig. 1-12 which result from maximum power theory. The following corollaries describe constraints that maximizing power places on models for any size dimension. Each corollary can be shown by energy circuit diagrams and these illustrations constitute Figs. 1-12.

(a) Energy must be upgraded and stored to accelerate inflow and effective use. See Fig. 1(b). When energy sources are above a critical minimum, the surviving systems must build and store energy of a higher quality, which is used as an autocatalytic positive feedback pump to accelerate the incoming energy flow. The systems that do this draw more power than those which do not have such pumps and have only their linear flows. The degradation of the incoming energy is faster and more competitive because of the upgrading of part of the energy. Storage is required in order to anchor delivery of high grade forces.

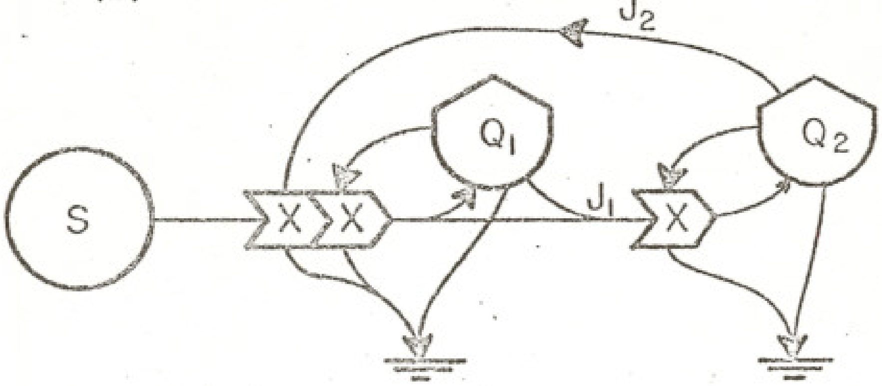
(b) Feedback work must balance drain in effect on energy. See Fig. 3. The required feedback work of high grade energy (J_2) must contribute as much effect toward augmenting upstream energy incoming to the unit as the unit is draining the energy source (J_1). If the feedback pathway does not return as

(a)



$$Q = (k_1 - k_2)QS - k_3Q$$

(b)



$$\dot{Q}_1 = kSQ_1Q_2 - k_2Q - k_3Q_1Q_2$$
$$\dot{Q}_2 = k_4Q_1Q_2 - k_5Q_2 - k_6SQ_1Q_2$$

Fig. 3. Diagrams showing reward loop feedback of upgraded energy quality acting upstream as amplifier so that a balance of payments of energy effect can be achieved between J_1 and J_2. (a) Balance of payments within one self regulating unit; (b) Balance of payments between two self regulating units. Q_2 greater quality than Q_1 which is greater quality than S.

much energy effect as it uses, it will stress its upstream source entity and thus weaken its competition for energy with others at that point upstream. Such arrangements will be competed against and eliminated.

(c) Net energy must be reinvested in energy gain. See Fig. 4. If a feedback generates more quality (Q) energy than needed to balance its energy cost, the excess high quality energy must be reinvested towards augmenting more energy out of new external sources (4b), out of the same source (4c), or in improving the effectiveness of the existing energy flows so as to diminish energy waste. Further energy storage may be part of the means to effective investment. A system that does not develop an autocatalytic reinvestment develops less energy and is eliminated.

(d) Energy storages must be of an upgraded higher quality. See Fig. 3. Since stored energy always has depreciation and losses inherent in its storage and its upgrading, it must be of a higher quality than its energy sources in order that the return reward loop which repays its drain should have amplifier value in pumping in new energy.

(e) Fifty percent power dissipation maximizes upgrading storage power. See Fig. 5. Storing energy of upgraded type at maximum power requires a back loading that neither stalls nor wastes input energy resources. There is a parabolic relationship between power output storage and loading [3 ; 7] which is a maximum at half storage and half dissipation. Greater back-loading slows power delivery; lesser backloading increases percentage power diverted.

(f) Energy quality is measured by energy transformation costs or by amplification ability. See Fig. 3a. The quality of energy is measured by the energy cost of its steady state generation by upgrading from an energy type of lesser quality. A pathway to be a surviving one has the effect of return flows

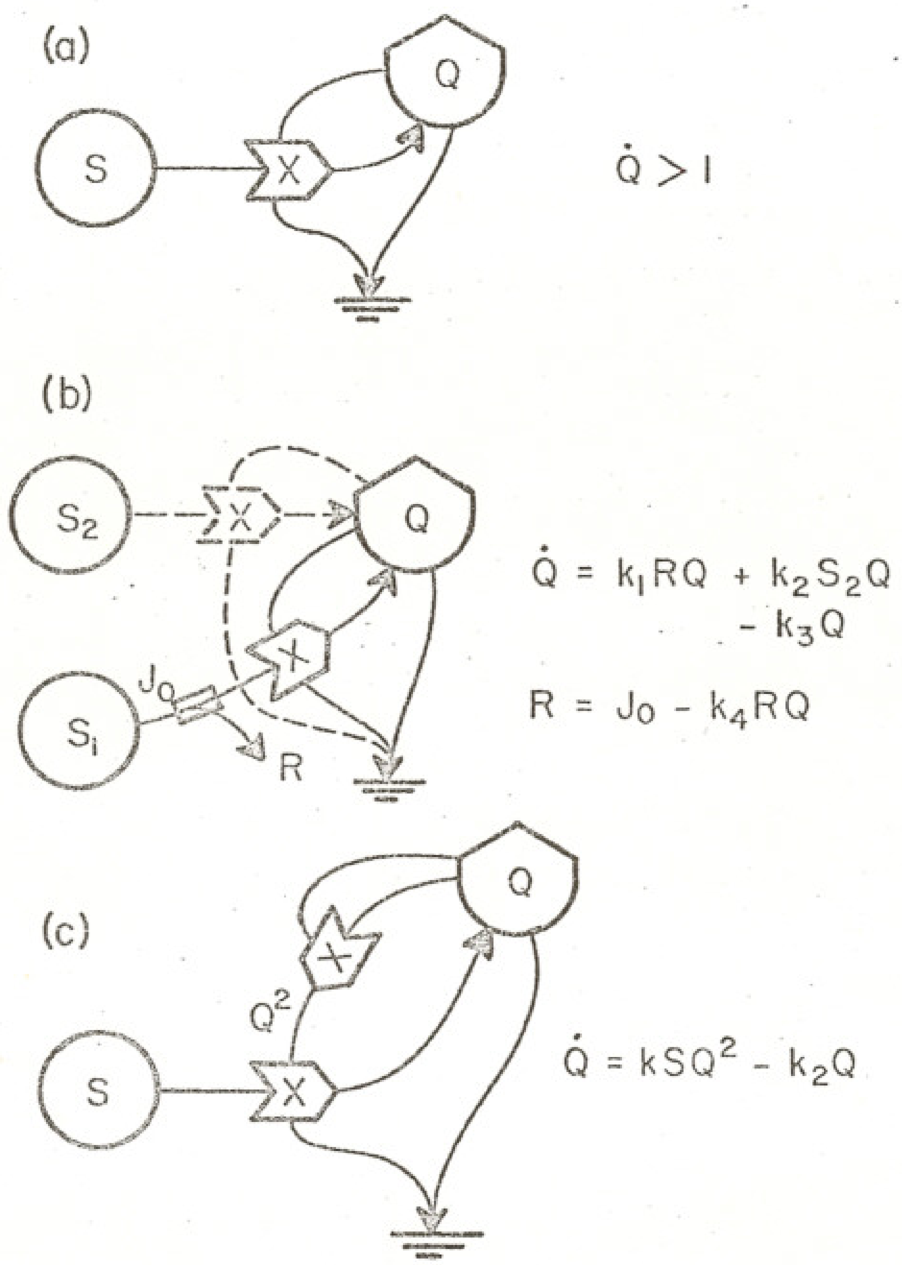
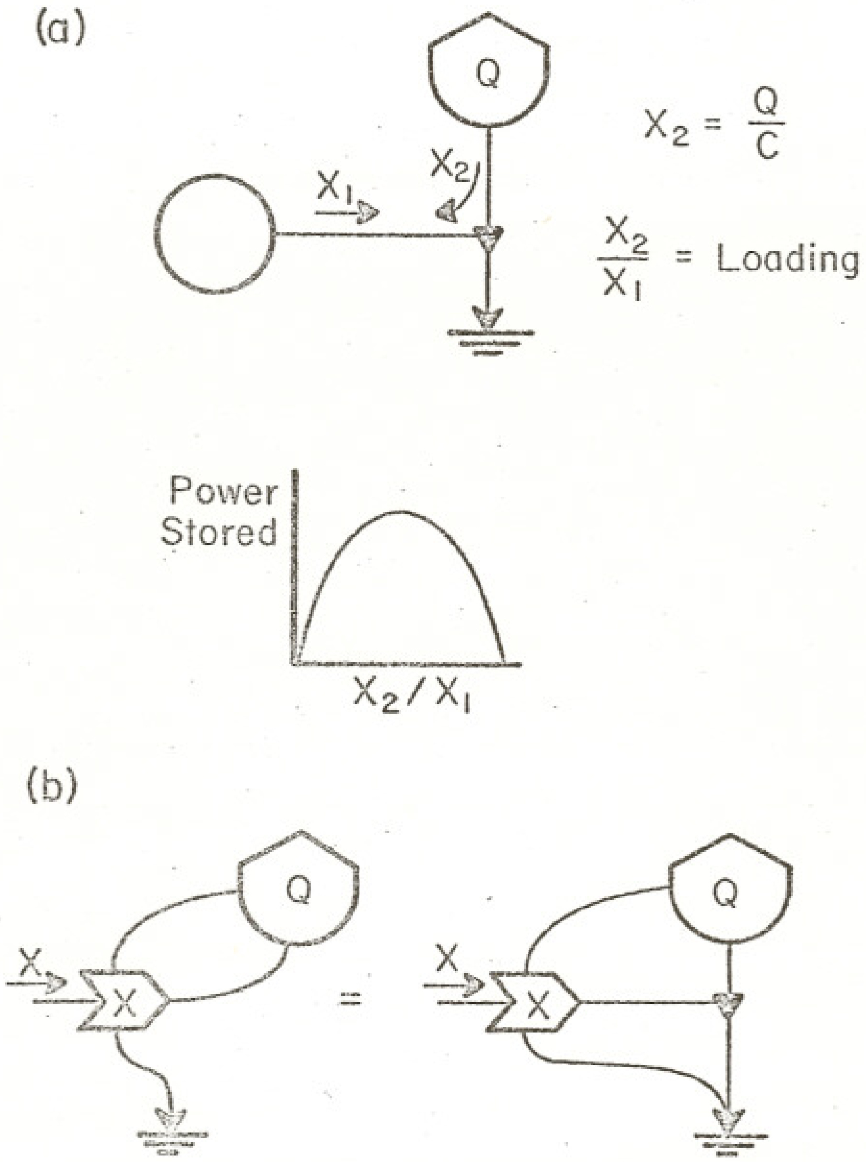


Fig. 4. Energy investment opportunities where Q is in excess. (a) Q grows augmenting J_1; (b) New process P initiated where S yields no further energy (c) Quadratic acceleration (Q^2).



$$\dot{Q} = (k_1 - k_2)\frac{Q}{C}\left(X - \frac{Q}{C}\right)$$

Fig. 5. Power and loading relationships for an upgrading and storage process. Notice that productive flow has no barb.

as great as their drain and thus feedback returns have to achieve amplification. The energy cost of the loop through upgrading to feedback amplification has its amplification ability measured by the energy dissipation to accomplish it including the storage maintenance and generation costs. Since selection tends to adjust amplifier loop effect equal to drain costs, ability to amplify is also a way to measure energy quality factor.

(g) Investment in growth is required if there are net energies and untapped energy resources. See Fig. 3. If energy supplies are untapped and rich enough so that there is a net accumulation of high quality energy storage, including balance of payments exchanges with surroundings (Fig. 3b), then accumulation (growth) is required since this provides stronger feedbacks with which to augment more energies. Premium on net growth strategy ceases when energy rate of supply cannot be increased because of source limitations (Fig. 4b).

(h) Growth efforts are wasteful when energy flows are source limited. See Fig. 4b. Attempts to pump in more energy from energy flows that are source limited and under full utilization do not succeed, and energy pathways of such investments generate no power reinforcement, whereas competing alternatives putting energies into more efficient mechanisms such as diversity of utilization or control may be more energy cost effective.

(i) All pathways above minimum energy threshold are non-linear. (Fig. 1) Because the maximizing of power requires the feedback pumping of a small amount of high grade energy interacting with a larger flow of energy concentration of lower grade, the input productive process must be an interaction such as a multiplicative one that supplies a limiting factor action (Fig. 1b). The feedback cannot interact by summing since this would be a waste of upgraded quality energy potential. Interactive flow develops more power than linear ones (Fig. 1a) except at very low energy source concentration.

(j) Power spectra due to energy quality chains. See Fig. 6. Maximizing total system power use requires the system to develop as many possible means possible for feedback enrichment requiring higher quality energies. Energy quality development is thus extended to second, tertiary, quarternary and longer loops that ultimately produce very high quality energy at the end but with very little quantity. Power selection develops chains of upgrading energy quality development. Since total energy is degraded along the chain, there is a percentage loss at each link in the chain. Thus exponential energy power spectra are developed. Examples are turbulent eddies, food chains, and chains of molecular energies of activation in molecular energy distribution. The balance of payments of feedback of this high quality energy provides special work that the primary upstream units could not do for themselves without the downstream chain. Maximizing power seems to account for the Maxwell-Boltzman distribution in the thermal world as well.

(k) Normal distribution may result from maximum power selection. If exponential power spectra are the result of the chain of energy quality upgrading (see corollary j), and in those cases in which the energy stored is as the square of a state variable (as kinetic energy is to velocity and electrical charge energy is to charge, and water elevation energy is to water quantity), there results a normal distribution in the state variables along the quality scale. In effect normal distributions result from power spectra, the latter resulting from the maximum power principle's selective action in generating energy quality chains.

(l) Quadratic energy investment may be required for survival of high energy. See Fig. 4c. Where energy sources are higher than some threshold, the energy

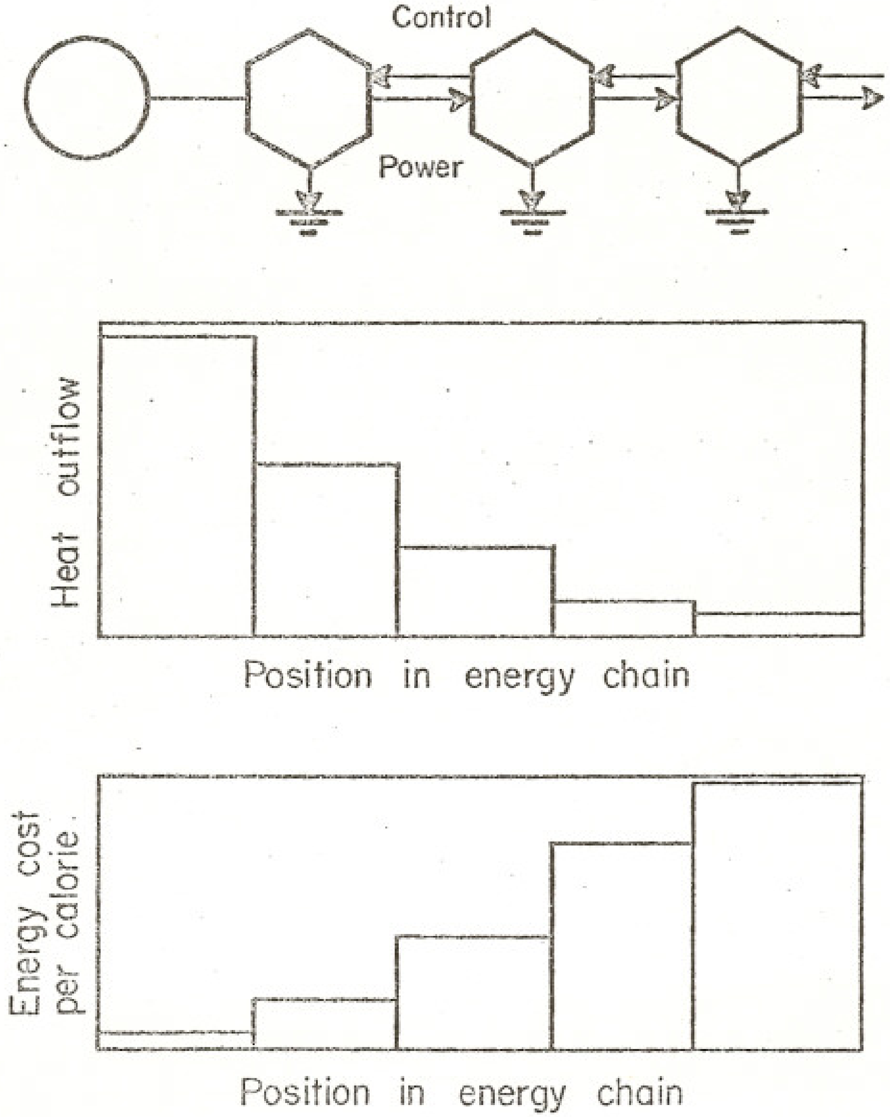


Fig. 6. Power spectra and energy quality scale developed by energy chain selected for maximum power and its requirements of balance of power.

to be obtained by feedback investment of stored energy in quadratic work functions exceeds that to be obtained by simple multiplicative pumping even though quadratic work is also more costly in energy drain from storages. Systems may develop varieties of quadratic pumping work which includes interactions of units, crowding, information processing, and diversity, all of which have quadratic energy costs for operation.

(m) Recycle of some materials is required and gives stability. See Fig. 7. Materials (space, nutrients, displaced populations, etc) which are not inflowing in excess need to be recycled at energy cost to the system to prevent power inhibition due to shortages. Such limited recycling materials provide a Michaelis-Menten kinetic effect so that increase in external energy sources of steady state has an asymptotic hyperbolic effect on output power. Increased energy cost of the faster cycling provides one of the limits to structure and function that follows from material shortage.

(n) All energies are resources; disordering energies are adaptive. See Fig. 8. All types of energy flow are harnessed by surviving systems. Energies that seem to disorder such as excess heat, gamma radiation, avalanches, etc. (D) induce selection for replacement of design so that the formerly stressing energy becomes a contribution to the ultimate system. Disordering is a necessary part of the continuous cycle of order and disorder since maintenance of complex structure is more cheaply done by redevelopment than repair. Urban renewal and fire in pine forests are examples of power promoting roles of disordering processes where systems are adapted. Disorders of turbulence are used as a stabilizing factor by inherently unstable plankton population systems that save energy by letting the mixing provide their stability. Disordering may increase diversity.

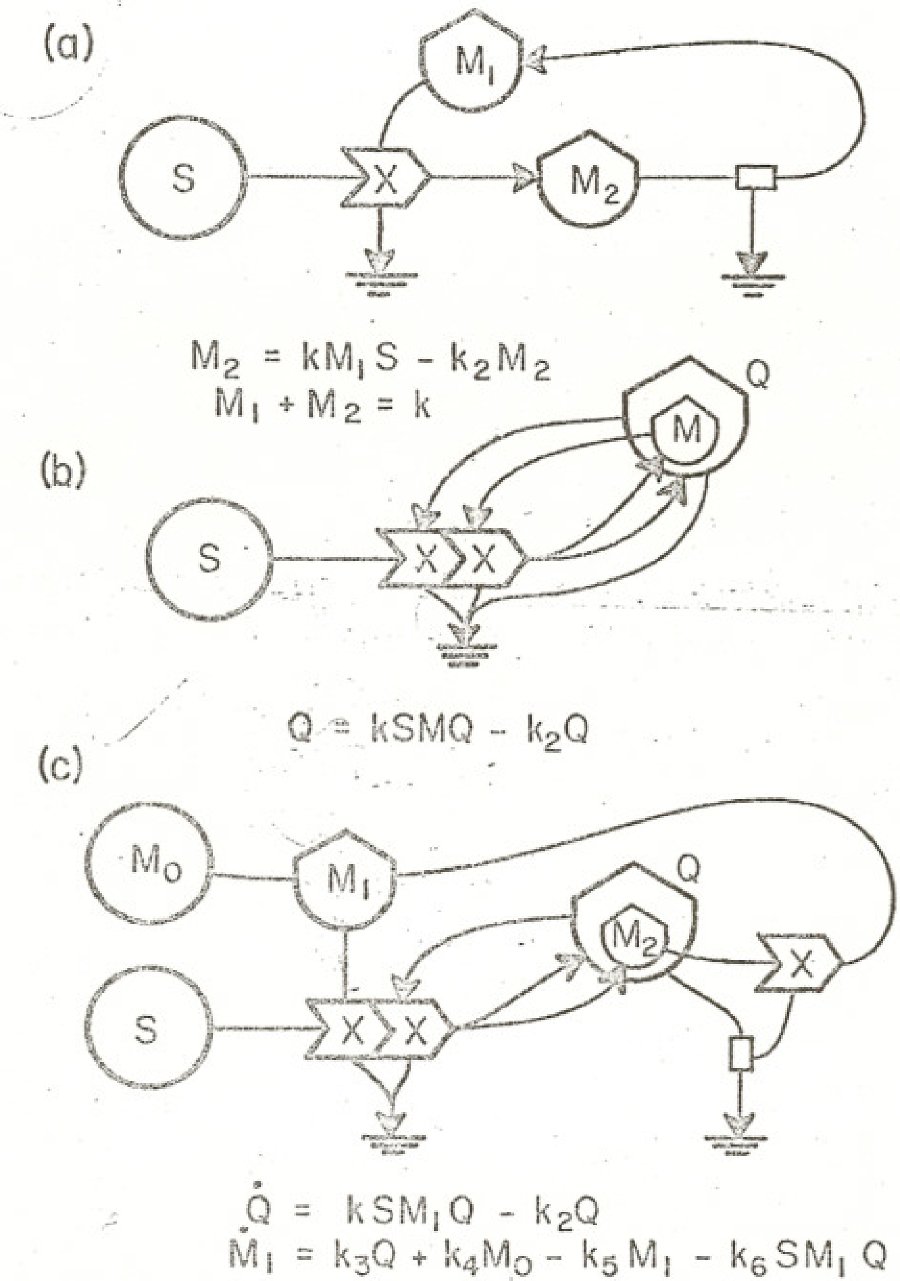


Fig. 7. Recycling matter isolated.

(a) Materials included as part of balance of energy exchange in autocatalytic model;
(b) Materials with special storage (M) in (Q).
(c) External material source and sink separate.

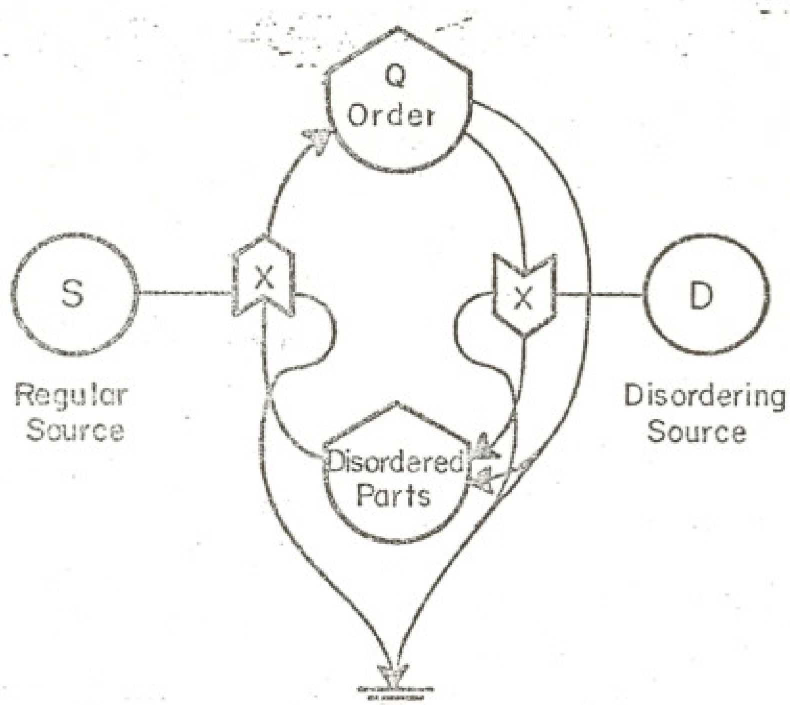


Fig. 8. Ordering and disordering energy source actions, both contributing to maximum power.

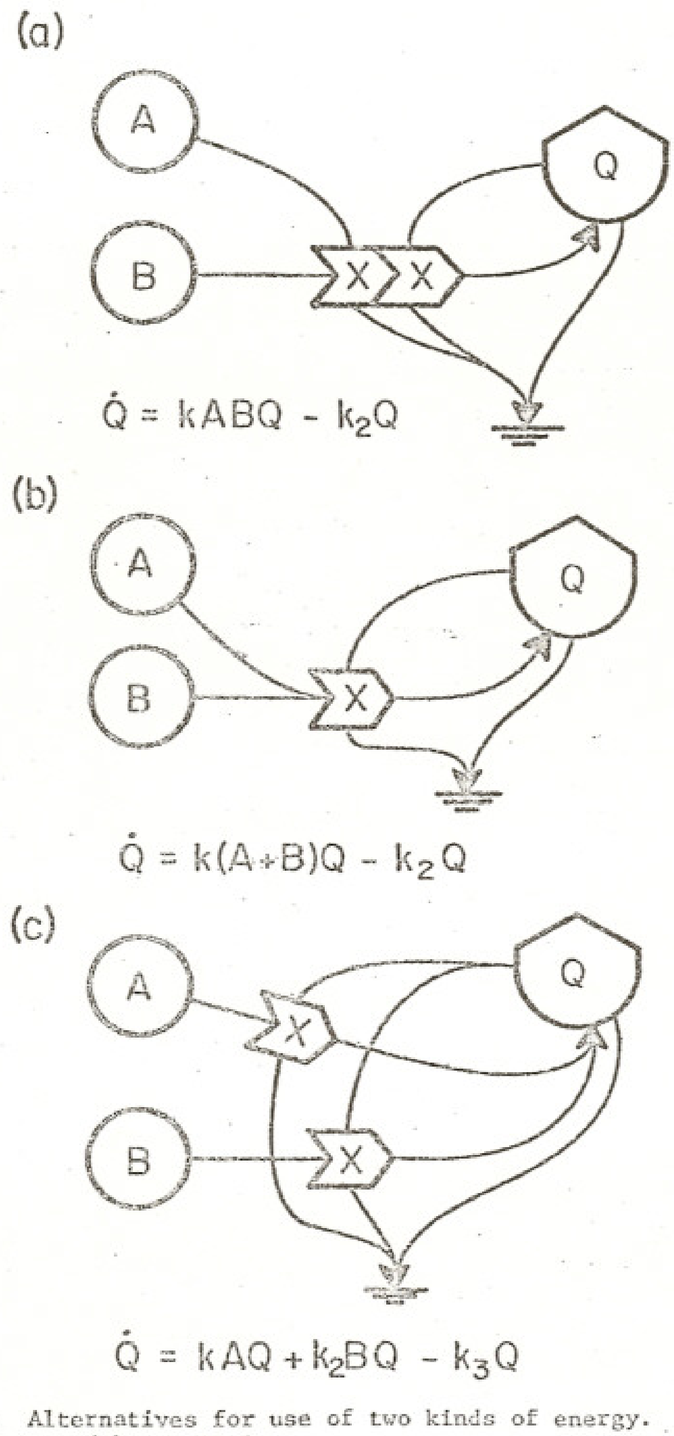


Fig. 9. Alternatives for use of two kinds of energy. (a) interactive (b) summing; (c) parallel.

(o) Energy resources of more than one type maximize power by interacting. See Fig. 9. If there are two kinds of energy, maximum power requires their interaction (9a) rather than their addition (b) or parallel action (c). Two types of energy are of two different qualities. One may serve as amplifier on the other, the high quality one augmenting the output of the other so that the useful power produced is more than the sum of the energies (heat equivalents not figured with quality factors). The best use of higher quality energy is as an amplifier on the lower quality energy which we often called a fuel. An example is the pumping (multiplication for some examples) of fuel by information activities and vice versa (d) see Fig. 11.

(p) Materials are stored only when external sources are irregular. Fig. 7c. The costs inherent in any storage require that only those materials that are in irregular or short supply externally be stored so that storage apparatus and drains be minimized. Ecosystems and their components avoid storing those materials that are large and steady in external source. Substitution of species with alternative mechanisms permits flexible adaptation of systems to the needed storage strategy.

(q) Economy of energy processing linear programs the ratios of specialists. See Fig. 10. Development of a range of parallel work specialists that use recycling materials in different ratio allows maximum power selection to linear program ratios of work components. Instead of competitive exclusion of one specialist by another the action of selection at the overall level retains the diversity of many specialists acting in parallel.

(r) Time and space diversity and information processing required. See Fig. 11. Where external energy sources (forcing functions) are varying maximizing power with time requires that information and diversity storage be

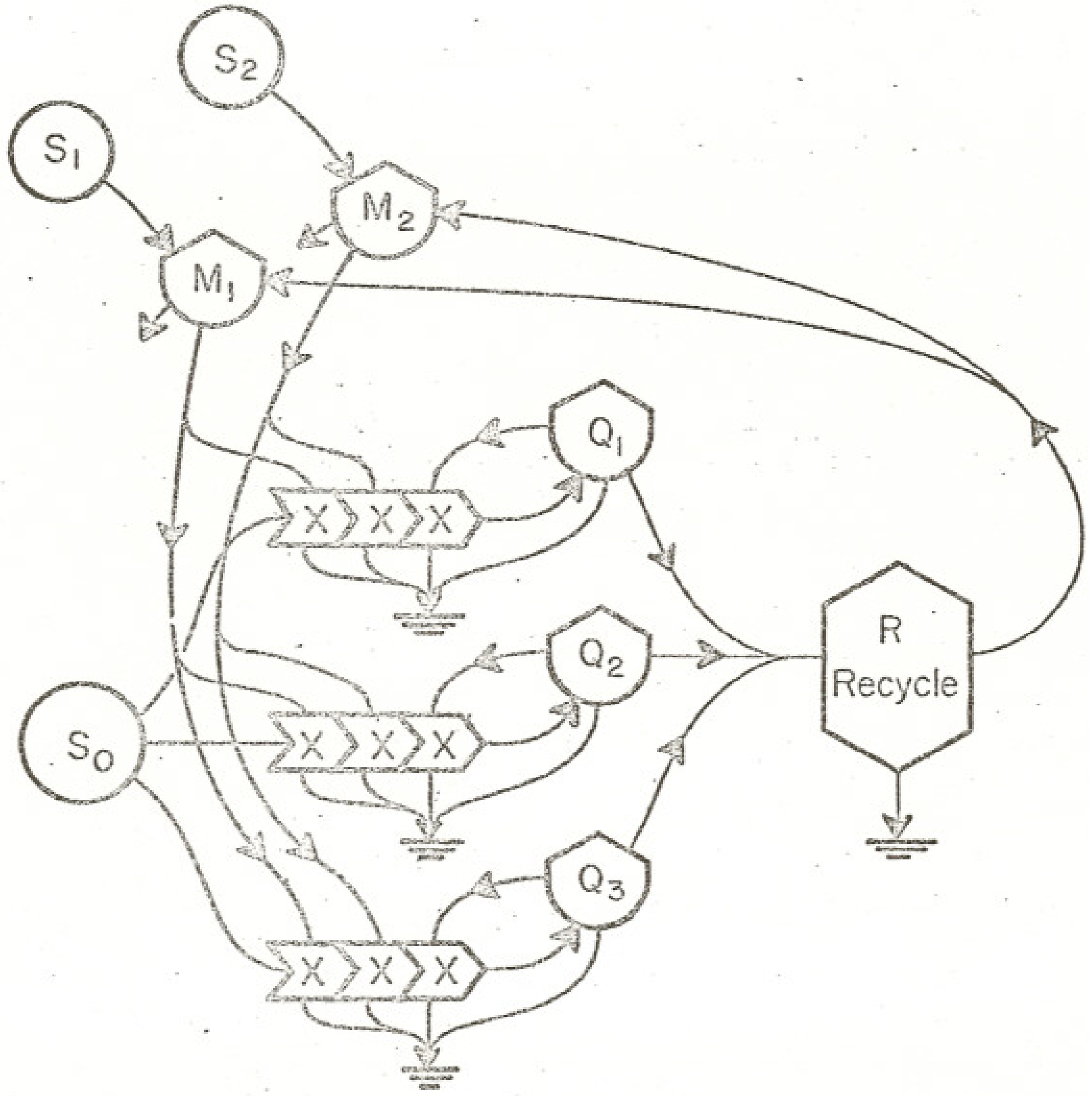


Fig. 10. Parallel units retained without competitive exclusion by systems self linear programming. See [2, p. 179].

organized for programming coupling responses to the energy regimes. This requirement for maximum power takes energy away from the concurrent diversity at any one time. Diversity with time and with space are complementary since they draw on common energy allocation for information processing and depend on the relative percent variation of energy inflows with time.

(s) Power and control circuits are differentiated. See Fig. 6 Pathways running downstream from energy sources are power circuits and those running back upstream are control circuits. Control pathways are high energy quality, high in information which tend to develop digital mechanisms whereas power pathways tend to be continuous functions and noise generators. Both are required and a system of flow for the two can be initiated by either, the one generating the other for autonomous survival.

(t) Energy flow develops choices that make power selection continuous. Alternatives are provided for power selection on all scale levels, since all energy flows develop side pathways, leakages, and variation. Sometimes we call energy diversion noise. However, disorder is the raw material of reordering that takes place by natural selection from the alternatives. Higher power generates more noise, choice, and factor selection. In the language a power spectrum of noise is assumed to be a part of each pathway and also a continuous generator of all possible pathways of units is also assumed.

Generalized Energy Constrained Model

In our discussion of corollaries, a number of pathways were found characteristic of any energy flow because of maximum power selection. Given in Fig. 12 is a summary diagram that includes all these together thus combining the

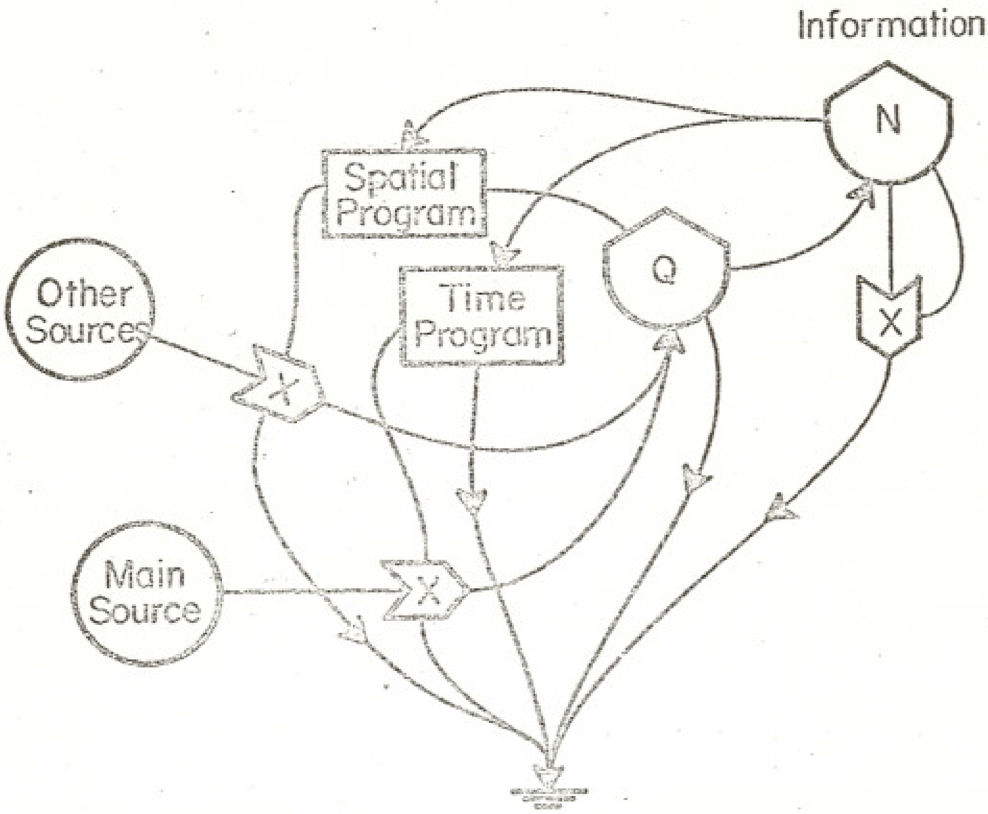


Fig. 11. Information, its quadratic energy cost and its alternative amplifier actions either in pathway diversity at one time or temporal energy adaptive mechanisms causing diversity with time.

corollaries of the maximum power principle. The differential equations for similar pathways are given also in Figs.1-9. Note how the translation in traditional mathematics is obscure and ambiguous. Energy circuits are readily learned and remembered visual mathematics, equally precise in meaning to traditional forms.

We have simulated many variations of this model and find that growth develops in time to an asymptote where the levels of the system either stabilize or in some cases have some oscillatory steady state properties. Wild unstable characteristics are eliminated if storages are large enough and energy drains are normal since both provide stability as already discussed. This energy constrained model has been elaborated for many real systems of interest such as populations of organisms, ecosystems, cities, the economy of the United States, the biosphere, and astronomical units such as stars and galaxies.

We issue a challenge to all mathematicians, scientists, engineers, and intellectuals of all backgrounds to join in the effort to make visual quantitative formulations useful, simple, and readily applicable. By combining the energy principles with other mathematics synthesizing the common visual diagrams, we may help to rally everyone about the simple visual language, which can be used in varying sophistication from the grammar school to the advanced think tank.

For lack of space here the inclusion of money and the manner by which power maximization drives the economy is omitted. See discussions elsewhere [2,5,6].

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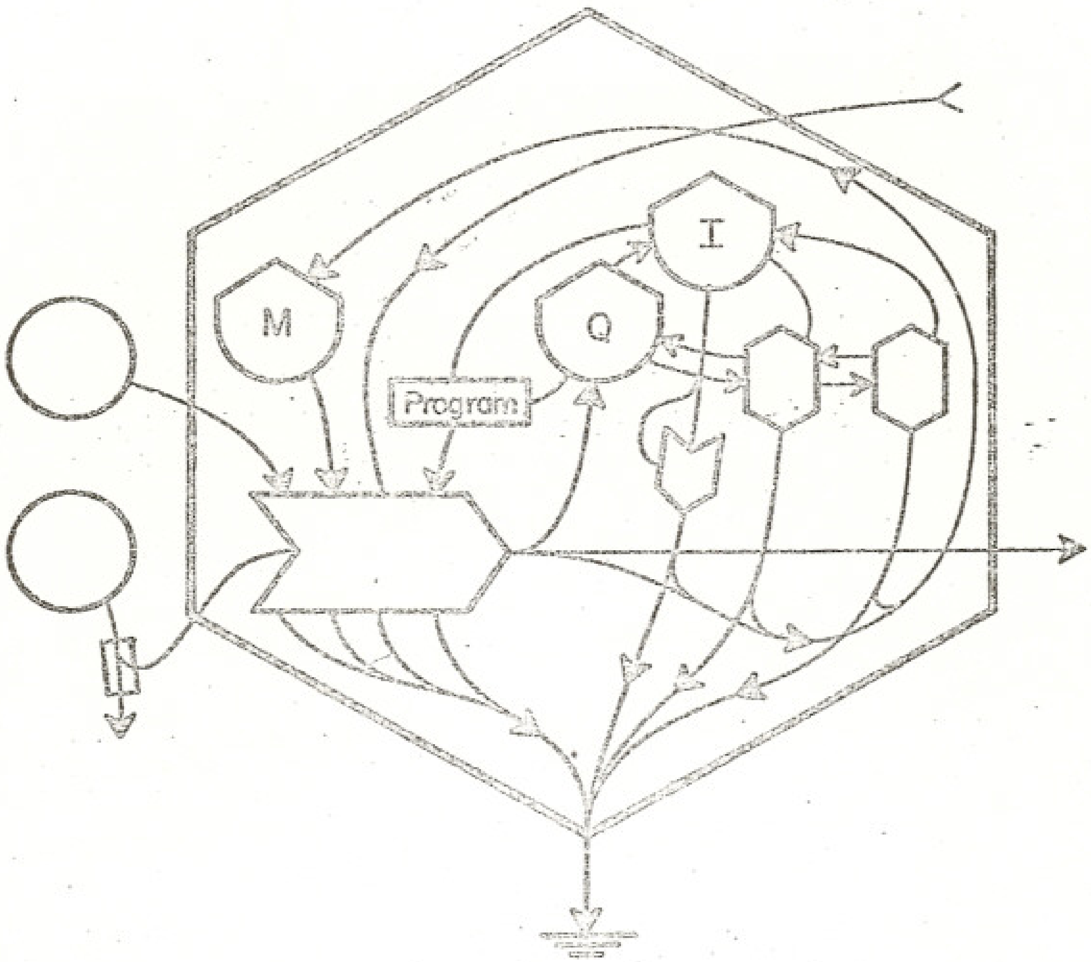

Fig. 12. Combination of some of the main pathways that tend to develop in adaptation of any system to maximum power selection. M, materials; Q, main storage; I, information storage. Equations follow.

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Bibliography

[1] Lotka, A.J. 1922. A Contribution to the Energetics of Evolution Proc. Natl. Acad. Sci. 8:147-155

[2] Odum, H.T. 1971. Environment, Power, and Society John Wiley, N.Y. 331 pp.

[3] Odum, H.T. 1972. An Energy Circuit Language, Its Physical Basis in a Systems Analysis and Simulation. Vol. 2, ed. by B. Patten, Adacemic Press

[4] Odum, H.T. 1973. Chemical Cycles with Energy Circuit Models pp. 223-259 in Changing Chemistry of the Ocean, Nobel Symposium No. 20 ed. by D. Dyrssen and D. Jagner, John Wiley, N.Y.

[5] Odum, H.T. Energy, Ecology, and Economics. 1973. Ambio 2:220-227

[6] Odum, H.T. 1975. Energy, Value, and Money. chapter in Models as Ecological Tools, ed. by C. Hall and J. Day in preparation; manuscript copy available from author.

[7] Odum, H.T. and R.C. Pinkerton 1955. Times Speed Regulator the Optimum Efficiengy for Maximum Power Output in Physical and Biological Systems, American Scientist 43:331-343.