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Short communication

MAXIMUM POWER AND EFFICIENCY: A REBUTTAL

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In a recent paper, Silvert (1982) criticizes our 1955 formulation of the relation of efficiency to power (Odum and Pinkerton, 1955) and our *times speed regulator principle* that selection of systems for maximum power regulates the efficiency at a value that is neither as large nor as small as loadings could operate a system. The energy that must be degraded to maximize power in a restorage process we called *entropy tax*.

Since there are other relevant literature, including our later discussions of power, efficiency and the maximum power principle, this note is supplied to correct and complement the discussion.

1. REBUTTAL OF DISCUSSION OF ATWOOD'S MACHINE

First let us dispense with Silvert's attack on the Atwood Machine: "The standard analysis based on Atwood's machine is not valid" and "The original attempt by Odum and Pinkerton (1955) to understand the bioenergetic functioning of animals was based on a misleading mechanical analogy and their mathematical analysis was flawed as well (Smith, 1976)".

Silvert's first comments are an example of the *straw dummy*: that is, set up a faulty assertion incorrectly attributed to others and then knock it over. The Atwood's machine was not the basis for our general derivation but was a special case used as an elementary teaching device to make the speed regulator principle clear to those not following the derivation. Whereas our derivation was for steady state, an Atwood's machine operated with a single drop is not a steady state. We were aware that the non-steady state Atwood's machine gives maximum power at 62%, and I had actually measured it experimentally in the physics laboratory at the University of Florida in 1953. Also, a letter to the editor of American Scientist pointed it out. If, however, an Atwood's machine were set up with a continuous loop so as to accept continuous input of weights and deliver continuous output of elevated

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weights, it would have the 50% conversion for maximum power, as we pointed out before.

Silvert said we used mechanical analogies to infer things about living systems. Incorrect; we used energetic formulations as the general case for which mechanical and living examples are special cases, each with special aspects. That was one purpose of the energy language (Odum, 1967; 1972).

2. WHEN IS THE EFFICIENCY FOR MAXIMUM POWER RESTORAGE 50%?

In our original derivation the efficiency for maximum power was shown to be dependent not only on the loading ratio but on the amount of energy dispersed in necessary coupled processes, which we called "leakage". Whether one gets exactly 50% or some lower percentage depends on whether one is considering a pure one step energy-storing transformation or whether the process has other necessary energy dispersals involved in the nature of the energy transformation over and beyond that necessary to restore energy of the same type.

Now that we recognize that real systems don't transform and restore energy without upgrading their quality (Odum, 1976) to a form or concentration that has greater feedback amplifier ability, it follows that there are almost always additional energy-using aspects to real transformations such as converging energy, storing it in a new form, etc. The graph originally given by us for the relation of efficiency of maximum power indicates a range of efficiencies that may be accompanying maximum power selection—depending on associated, energy-diverting processes.

Tribus (1961) took our derivation, confirmed its validity, and did a better job of presenting it in thermodynamic terms with a useful nomogram. In my paper on power concepts built into the energy language (Odum, 1972), I gave our 1955 derivation without the leakage in order to isolate the loading effect alone. See also Odum (1983). The efficiencies of any observed process are those of a system of energy coupled inputs and outputs that are best described with a network diagram, whereas the 50% is only for the single step loading without the other energy uses or leakages of a system in which it is embedded.

Fifty percent loadings were observed for single processes. Milsum (1966), for example, used data on muscle to show optimum efficiency for maximum power of muscles lifting weights near 50% (although this may not include all the energy involved in the organismic systems supporting the muscle). See also a different example of muscle (Odum, 1983). However, most energy uses involve more than a single step energy restorage.

We studied power and efficiency in bluegreen algal mats and their electrical output (Armstrong and Odum, 1964) and found a parabolic loading curve as predicted by theory, as did Billig and Plessner (1949) with solar voltaic cells. Efficiencies were far less than 50% and energy transformed included several steps and a large upgrading of energy quality.

In some elegant work, a combination of theoretical formulations and measurements of loading and power transfer were made in biochemical processes, membranes, etc., by Caplan (1966 and later papers). In a paper on photosynthetic production, the first steps of photosynthesis as observed in Hill reactions were found to have an optimum efficiency for maximum power loading (Odum, 1968; 1983).

3. EFFICIENCY AND MULTIPLE PROCESSES

An efficiency is the ratio of any two energy flows. On a typical diagram of a system, depending on how much detail is aggregated, there may be dozens of efficiencies, some including others. The more one aggregates, the more successive storage, restorage and other kinds of processes may be included, and the lower will be the efficiency for maximum power. Organisms and ecosystems generally have multiples of processes, each of which has its optimum efficiency for its maximum power transformation, and contributes to an overall optimum efficiency for maximum power. As said before, the 50% is not expected unless one has isolated a transformation (or studies a process predominated by a single transformation) that is pure restorage, without leakage, without connecting processes, and without quality upgrading. That most systems are complex does not make the principle about entropy tax and an inherent loading for survival any less correct.

Another complication is restorage of energy without transformation. Fat ingestion and storage is an example. This energy transfer may cause efficiencies higher than 50%.

4. LINEARITY AND NON LINEARITY

Much of the irreversible thermodynamics that considers coupling of processes has been on linear processes and our derivation was for energy restorage with linear coupling of forward and back forces. Classical equilibrium thermodynamics, being static, does not worry as to the nature of pathways since there is no net process. Relationships are independent of pathways.

Open system thermodynamics involves energy transformations in pathways, and it might be argued, as Silvert does, that the presence of a complexity of non-linear pathways makes a derivation for linear processes irrelevant. To what extent irreversible thermodynamics is pathway dependent is still perhaps an open question. It is certainly a thermodynamic ideal that useful energy formulations be independent of path. Many systems of non-linear processes become linear at steady state. It has been an additional postulate by us (Odum, 1967) that after selection, surviving systems have organized their pathways and loading so as the minimize entropy tax at maximum power. If this is true, this provision selects whatever pathway makes the thermodynamics criteria maximum. If true, the pathway becomes a dependent property and Silvert's criticism is irrelevant. In other words, power is fed back to provide the best pathway possible thermodynamically. Whether the maximum power loading is predicted by the linear case derivation is still an open question.

5. MAXIMUM POWER EFFICIENCY IN OTHER KINDS OF PROCESSES

Whereas our first derivation was for the single step transformation of one energy flow, using potential energy restoring energy, it was pointed out that other kinds of processes also had an optimum efficiency for maximum power transfer. Impedance matching involves the matching of dissipative loads so as to minimize losses that would reduce power conversion where one of the dissipative loads is useful. A useful process can be defined as one that feeds back work to the supporting or surrounding systems. A grindstone is dissipative, but upgrades the energy of food to a higher quality, usable by humans for example. Organizing fishes into a school is dissipative (swimming), but the product has useful adaptive values to the fishes and to the ecosystem. This principle is widely used in electronics (impedance matching).

Energy dissipated in organization also has an optimum for maximum power where the power output is a dissipative process (Odum, 1972).

6. ENERGY TRANSFORMATIONS WITHOUT ENTROPY TAX

About 1959, B. Strehler posed the question to me, why electrical transformers and pendulums stored and restored energy without much dissipation and seemed to be an exception to the times speed regulator principle. The answer to this was found from Einstein (1905), that processes that involve potential to inertial transformations (e.g. acceleration of mass in a pendulum or acceleration of electons in a transformer) are symmetrical, depending on whether the observer is moving or stationary, and thus are only motion-relative energy restorages and have no second law energy loss. Such energy conversions have traditionally been covered by the Hamiltonian law, which minimizes the Lagrangian integral of difference between the kinetic and potential energy. In one sense this maximizes power transformation.

7. INDEPENDENT FORMULATIONS

Apparently the optimum efficiency for maximum power principle has been independent formulated many times by those not aware of the papers of others. As we did in 1955, Curzon and Ahlborn (1975) consider the efficiency of heat engines at maximum power with the same result, which they found agreed with observed power plants. The reason that efficiency of power plants vary, is that the auxiliary leakages and the Carnot ratio both vary with temperature difference. See also Andresen et al. (1971).

Still other initiatives, apparently independent, are those by Fairen and Ross (1981); Fairen, Hatee and Ross (1982). These papers have other interesting formations as well as dealing with maximum entropy production and speed. (Maximum entropy productions is another way of referring to maximum power utilization if feedbacks couple the products of power use to power generation.)

8. ECONOMIC SYSTEM MAXIMIZING POWER

It is a postulate (Odum, 1971) that the free market economic system maximizes power of the system-constraining components to this aim, even though economics has generally evolved with the idea that the human final demand consumer is free to choose the nature of the network. Consider power plants. Power plants are selected by the engineers, and the economic system in which they are embedded, for maximum power delivery and thus they are adjusted by trial and error to the optimum efficiency. Here, humans are the instrument of energy law operating on the large-scale system of man and nature. In other words, maximum power principle operates on human decision without humans being aware of it. The individual human thinks it is human free will as he tries this or that, but what the system selects and uses because of its power characteristics follows the times speed regulator corollary of the maximum power principle. Social institutions, political processes and economic analyses facilitate group decisions to continue programs that work, i.e. maximize power.

In an independent formulation of the role of maximum power in maximizing money flow, Wesley (1974) gives derivations with graphs of power efficiency and profit for the case of thermal engines (typical of our economic base) driving money loops.

9. MAXIMUM POWER SELECTION AND YIELD

That there is an optimum efficiency for maximum net yield of a population in exponential or logistic growth is as well known as optimal catch in biological sciences. See for example, the Schaeffer model (1954) or the yield of dollars based on operating optimum catch for maximum profit (Clark, 1973). When based on the logistic model, these are based on a non-linear model, but one which is linear at steady state. If one considers, as the system of concern, the fishery population *and* the fishery (with its energy source), then operating at maximum yields and profits maximizes power utilization for that model and is probably another special case of the maximum power principle.

It is not a good model for the real world, as fishery people are realizing, because it drains its supporting food chain without a feedback amplifier, thus causing its replacement by other food chain elements. What is maximized is the power of the larger system, which the fishery yield models did not do.

10. BIAS AGAINST RATE PRINCIPLES

In the 1950s and 1960s Pinkerton * and I used to discuss why the times speed regulator and maximum power principles were being ignored in both biological and physical fields (now no longer true) when it seemed to eliminate much of the wild freedom with which the world's systems were being considered. Our 1955 paper was at first rejected, until K.E. Denbigh wrote a letter in its support. Many physical scientists are not trained in natural selection and think of natural selection as tautological because it involves a closed loop. Partly it was a question of fear. Many did not want their fields to have such a constraining law that would simplify and reduce the mystery and value of their careers of measurement. Those with heavy statistical views believed nature to be basically indeterminate. Application to human-scale systems and economics was counter to the roots of these fields in the dogma of humanism, where the indeterminate and free will judge of value is the central belief. Energy determinism in the fifties and sixties implied limits to growth imposed in a world where growth was identified with progress. It was easy for emotional fears of opposite kinds to form a majority coalition to riducule and blacklist the maximum power approach and discredit search for general laws that might make some studies unnecessary.

* We miss the spunky independent clarity of R.C. Pinkerton, whose career was shortened by his untimely death in 1966.

11. SYSTEMS SCALE AND MAXIMUM POWER

Partly it was a problem of system scale. The concept that there is a natural selection for maximum power was viewed differently by those working on one scale from those working on another, whereas the principle applies simultaneously to all size scales.

For example, those working with a population of birds thought that selection for maximum power meant selection for maximum energy flow to birds. Selection for maximum power does not mean a selfish maximizing of power by components. Components maximize their survival by contributing to the power of the whole supporting system by means of feedback works and by converging service to the center of surrounding larger systems so that their life support system maximizes power. Low energy specialists, for example, maximize system power by filling in the use of energy potentials missed by more dominant components. Their own storage efficiency may be higher than 50% if operating at lower speed (maximizing system power at a higher priority than maximizing power of its own process).

12. "LOOK SMALL" TYPE OF REDUCTIONISM

Scientists at one level, through preoccupation at that level and absence of course tranining of science at the next larger size, often deny that the next larger system is their concern and sometimes even deny that there is any science there (denying organization and control actions that might be setting the nature of the systems they study). In the hierarchy of nature no level is independant of either its part or its part in the next, but the education of our century, using a perversion of the word "basic", has looked mainly to smaller realms. Being trained in this way, it was natural to misunderstand to what survival of the fittest in regard to maximum energy use referred. Actually, it refers to the larger and smaller scales both, so that the designs that are developed are those that couple these systems to maximize the total power.

Silvert's discussion is typical of those considering only the species when he discusses the efficiencies of marine organisms in food conversion as being fairly similar and concludes that optimum efficiency is, after all, "broadly applicable—and a major contribution to ecological theory". According to the systems view mentioned before the observed efficiencies cannot be appraised by themselves. The appropriate measurement is the power utilization of the system in which the species and their conversions are contributing their services. Only in the context of the larger system can one determine whether a given species job is generating net production, or generating a

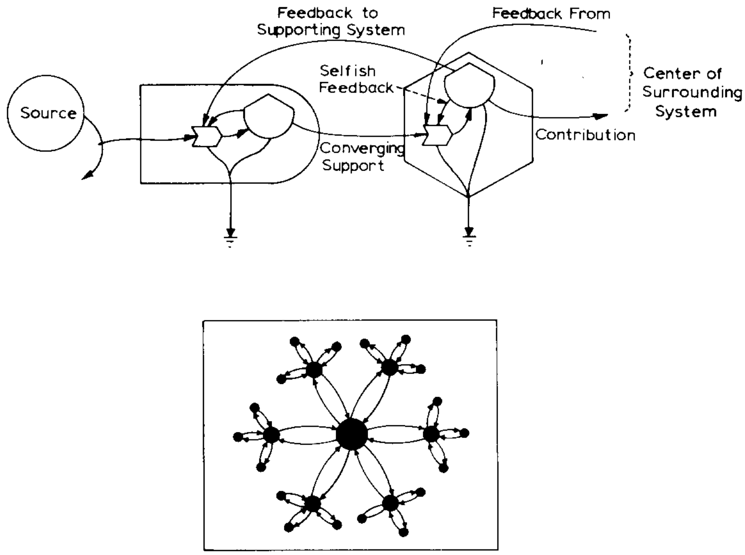

Fig. 1. Diagram of a three stage ecosystem hierarchy with feedback to help maximize power of the whole network. (a) Energy language diagram; (b) spatial pattern of converging energy use and diverging feedbacks.

non-stored service, or processing information, or being usefully dissipative as part of larger system organization.

Those who have discussed group selection think that survival of the fittest refers to the strategies for maintaining the group as a competitor with other species and groups. This has very little to do with self organization, which uses pools of species variety to form larger systems that reinforce, through feedbacks, to maximize power. Here the selection is among alternative organizational connections, nutrient cycles, feedback controls, behaviors, etc., because they generate a reward loop for augmenting power within the system. Many who study strategy often think of the strategy that is selfish, whereas the highest selfishness is the strategy that maximizes the next larger system, thus insuring self existence through maintaining habitat.

Figure 1 shows the way a unit of intermediate size contributes to its smaller contributors and to its larger controller, as well as to itself, so that the network maximizes power.

13. MAXIMUM UNUSEFUL POWER

Another misunderstanding of maximum power principle is that the principle means using energy indiscriminately. The statements by Lotka (1922a and b) and our many corollaries since (Odum, 1975) mean "useful power" where "use" is feedback of the product of energy use to amplify other pathways. As used in engineering, power means rate of useful work.

14. TIME FOR MAXIMIZING POWER

Rather than considering power in a constant type of steady state, it is becoming apparent that pulsing alternation of production and consumption may be the patterns which maximize power and long range survival (Odum, 1979; Richardson and Odum, 1981; Odum, 1982), where energies are in a certain range. A general proof is needed.

15. OTHER OBJECTIVE FUNCTIONS

Many criteria have been suggested besides maximum useful power as the self design criteria of ecosystems, e.g.: maximum efficiency, maximum entropy, maximum diversity, maximum reproduction, maximum biomass, minimum entropy generation, maximum profit, etc. There are instances in which each of these criteria may be optimized so as to favor the survival and continuation of a system, but I believe all of them to be special cases of the maximum power selection principle. For example, Prigogine and Wiaume's (1946) early use of minimum entropy generation as a design criteria is now agreed by him and by us (see recent symposium: Mitsch et al., 1981) to apply only at very low energies not capable of supporting such autocatalytic systems as life and turbulence. For another example, Margalef (1963) and Odum (1969) suggest that more biomass can be obtained per unit metabolism with mature systems. This seems to me to be an inverted view of the property that there are diminishing returns in developing biomass to maximize power. Beyond a certain quantity, adding biomass does not increase power, but other improvements in design, such as diversity, then become more important.

Fontaine (1981) devised a model of the self organizing process for the study of ecosystem patterns that result from various objective functions. Maximum power generated the observed patterns.

Odum (1972) and Sugiyama and Shimazu (1972) extrapolated the power–efficiency derivation to provide an efficiency-loading nomogram. Sugiyama and Shimazu suggest that ecosystems maximize efficiency at a different time from power in succession. One point of confusion is the difference between net production and gross production. Maximum power corresponds to gross production (if there are no fuel inputs). During growth, maximum power goes to maximum net storage. Later, power is often maximized in mature states by connecting all net production to diversity and useful service. If there is a source from non-renewable storage, power is maximized by rapidly using up the storage and then operates at a lower level maximum for the renewable source. We believe there is an optimum frequency of net storage and use in pulses that maximizes long range power (Odum, 1979; 1983; Richardson and Odum, 1981).

A much fuller discussion of all these is given in a book, although the arguments are dispersed through a long text (Odum, 1983).

16. HISTORY AND COROLLARIES OF MAXIMUM POWER SELECTION

The concept that maximum power is the basis for economies of nature and of humanity has many roots, such as the writing of Boltzmann (1905) and Ostwald (1907) before Lotka (1922a, 1922b) and a long series since (see historical statements, Odum, 1982b). We find many corollaries as to the design characteristics of surviving systems that follow from the maximum power and speed regulator principles (Odum, 1967, 1972, 1976, 1982, 1983; Odum and Odum, 1976, 1981). There are many predictions about the changing economies of the world that seem to be becoming verified. That there is a general design form for ecosystems and economic systems with behavior traceable to this principle is, to me, greater proof of the generality of the maximum power concepts, rather than the efficiency measurements discussed by Silvert which really concern what percentage of energy used by a species is producing a service or a stored product.

In summary, putting his straw dummies aside, we should be grateful to Silvert for coming to grips with one aspect of times speed regulator, for indicating the need for a general non-linear formulation of power and efficiency, and for providing a dialogue to reaffirm the connections between the energetics of ecosystems and the design of component interrelations. It was Lotka who suggested that the maximum power principle should be regarded as the fourth law of thermodynamics (also Odum, 1963). It seems to be well on its way to that status.

REFERENCES

Andresen, B., Salamon, P. and Berry, R.S., 1971. Thermodynamics in finite time, extremes for imperfect heat engines. J. Chem. Phys., 66: 1571–1577.

Armstrong, N.E. and Odum, N.T., 1964. Photoelectric ecosystem. Science, 143: 256–258.

Billig, E. and Plessner, K.W., 1949. The efficiency of the selenium barrier-photocell when used as a converter of light into electrical energy. Philos. Mag., 40: 568–572.

Boltzmann, L., 1905. The second law of thermodynamics. Populaire Schriften. Essay No. 3 (address to imperial academy of science in 1886). Reprinted in English in Theoretical Physics and Philosophical Problems. Selected writings of L. Boltzmann. D. Reidel, Dordrecht, Holland.

Caplan, S.R., 1966. The degree of coupling and its relation to efficiency of energy conversion in multiple flow systems. J. Theor. Biol., 10: 209–235.

Clark, C.W., 1973. The economics of overexploitation. Science, 181: 630–634.

Curzon, F.L. and Ahlborn, B., 1975. Efficiency of a Carnot engine at maximum power output. Am. J. Phys., 43: 22–24.

Einstein, A., 1905. On the electrodynamics of moving bodies. In: H.A. Lorentz, A. Einstein, H. Minkowski and H. Weyl (Editors), The Principles of Relativity. Dover Publ., NY.

Fairen, V., Hatee, M.D. and Ross, J., 1982. Thermodynamic processes, time scales and entropy production. J. Phys. Chem., pp. 70–73.

Fairen, V. and Ross, J., 1981. On the efficiency of thermal engines with power output. J. Chem. Phys., 75 (11): 5490–5496.

Fontaine, T.D., 1981. A self-designing model for testing hypotheses of ecosystem development. In: Progress in Ecological Engineering and Management by Mathematical Modelling. Prov. 2nd Int. Conf. on State-of-the-Art, Ecol. Modelling, Copenhagen. pp. 281–291.

Lotka, A.J., 1922a. Contribution to the energetics of evolution. Proc. Natl. Acad. Sci., 8: 147–155.

Lotka, A.J., 1922b. Natural selection as a physical principle. Proc. Natl. Acad. Sci., 8: 151–154.

Margalef, R., 1963. On certain unifying principles in ecology. Am. Naturalist, 97: 357–374.

Milsum, J.H., 1966. Biological Control Systems Analysis. McGraw Hill, NY, 466 pp.

Mitsch, W.J., Ragade, R.K., Bosserman, R.W. and Dillon, J.A., Jr., 1981. Energetics and Systems. Ann Arbor Science, Ann Arbor, MI.

Odum, E.P., 1969. Strategy of ecosystems development. Science, 164: 262–269.

Odum, H.T., 1963. Limits of remote ecosystems containing man. Am. Biol. Teacher, 25: 429–443.

Odum, H.T., 1967. Biological circuits and the marine systems of Texas. In Burgess and Olsen (Editors), Pollution and Marine Ecology. John Wiley, NY, pp. 99–157.

Odum, H.T., 1968. Work circuits and system stress. In: H.E. Young (Editor), Symposium on primary productivity and mineral cycling in natural ecosystems. Univ. of Main Press, ME, pp. 81–138.

Odum, H.T., 1971. Environment, Power and Society. Wiley-Interscience, NY, 336 pp.

Odum, H.T., 1972. An energy circuit language for ecological and social systems: its physical basis. In: B. Patten (Editor), Systems Analysis and Simulation in Ecology, Vol. 2. Academic Press, NY, pp. 139–211.

Odum, H.T., 1975. Combining energy laws and corrollaries of the maximum power principle with visual systems mathematics. In: Ecosystem Analysis and Prediction. Proc. Conf. Ecosystems. SIAM Institute for Mathematics and Society, pp. 239–263.

Odum, H.T., 1976. Energy quality and carrying capacity of the earth. (Response at Prize Ceremony, Institute de la Vie, Paris). Trop. Ecology, 16 (1): 1–8.

Odum, H.T., 1979. Energy quality control of ecosystem design. In: R.F. Dame (Editor), Marsh Estuarine Systems Simulation. Belle W. Baruch Laboratory in Marine Science, Univ. of S.C. Press, CA, pp. 221–235.

Odum, H.T., 1982. Pulsing, power and hierarchy. In: W.J. Mitsch, R.K. Ragade, R.W. Bosserman and J.A. Dillon, Jr. (Editors), Energy and Systems. Ann Arbor Science, Ann Arbor, MI, pp. 33–59.

Odum, H.T., 1983. Systems Ecology. John Wiley, NY, 644 pp.

Odum, H.T. and Odum, E.C., 1976, 1981. Energy Basis for Man and Nature. McGraw Hill, NY, 331 pp.

Odum, H.T. and Pinkerton, R.C., 1955. Time's speed regulator: the optimum efficiency for maximum power output in physical and biological systems. Am. Sci., 43: 321–343.

Ostwald, W., 1907. The modern theory of energetics. Monist, 17: 511.

Prigogine, E. and Wiaume, J.M., 1946. Biology et thermodynamique des phenomenes irreversibles. Experiantia, 2: 451–453.

Richardson, J.R. and Odum, H.T., 1981. Power and a pulsing production model. In: W.J. Mitsch, R.W. Bosserman and J.M. Klopatek (Editors), Energy and Ecological Modelling. Elsevier, Amsterdam, pp. 641–648.

Schaeffer, M.B., 1954. Some aspects of the dynamics of populations important to the management of the commercial marine fisheries. Inter-American Tropical Tuna Commission Bulletin, 1(2): 27–56.

Silvert, W., 1982. The theory of power and efficiency in ecology. Ecol. Modelling, 15: 159–164.

Smith, C.C., 1976. When and how much to reproduce, the tradeoff between power and efficiency. Am. Zool., 16; 763–764.

Sugiyama, K. and Shimazu, Y., 1972. Some problems in economy oriented environmentology. Technical Environmentology I. J. Earth Sci., Nagoya Univ., 20: 1–29.

Tribus, M., 1961. Thermostatics and Thermodynamics. D. Van Nostrand, Princeton, NJ, 641 pp.