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Calculating solar equivalence ratios of the four major heat-producing radiogenic isotopes in the Earth's crust and mantle

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ABSTRACT

As part of the ongoing work in defining a consistent and unified geobiosphere emergy baseline (GEB) this paper considers the radiogenic component of the available energy from geothermal sources (one third of the global tripartite: solar radiation, dissipation of tidal momentum, and geothermal exergy). Recent literature suggests that Earth's geothermal energy results from two very different sources, decay of radioisotopes and primordial heat (heat left from Earth's accretion). In previous baseline computations, the radiogenic component of geothermal exergy was added to primordial heat, given various names like "deep earth heat", and a single transformity was computed for the combined sources. With the acknowledgment that the geothermal component of the GEB had two different sources, it became apparent that a single transformity may no longer be appropriate, thus a method of computing separate transformities was necessary. In a novel approach, this paper uses gravity as the primary input to both solar radiation and heavy radionuclides and computes gravitational transformities for both. Then solar equivalence ratios (SERs) are computed between solar radiation and the four major crustal radionuclides (^{238}U, ^{235}U, ^{232}Th, ^{40}K). The SERs are combined with published radiogenic geothermal exergy data to calculate the solar equivalent exergy of the radiogenic component of the geothermal flux. This equivalence method can be used to derive a theoretically and methodologically consistent calculation for the other inputs to the global emergy baseline (i.e. tides and primordial geothermal heat flux) that can be similarly quantified in terms of gravitational exergy required to produce them.

1. Introduction

The ongoing work of defining a unified geobiosphere emergy baseline (GEB) has resulted in several key findings that are shaping our understanding of the sources, processes and flows of available energy that drive the geobiosphere. Early on, Raugei (2013) made two assertions that are fundamental to this current work. The first was that "...two exergy flows which were clearly produced by different processes (such as RadHeat (*radiogenic heat*) vs. CrustHeat (*crustal heat*) ...) should not be expected to have the same transformity..." The second in the same paper (Raugei, 2013) was "...since the origins of tidal exergy and 'deep earth heat' cannot be traced back to solar radiation, it is arguably conceptually impossible to compute solar transformities for them..." Raugei's first assertion resulted in our search for distinguishing radiogenic heat from primordial heat and then searching for methods of computing solar equivalences. The second assertion had profound effects on how that computation should be done, and ultimately on the terminology used to describe the results of that computation.

We must deal with the second assertion first in order to make sense of the terminology we will use to discuss the first assertion and for the rest of this paper. Raugei (2013) rightly observed that solar energy in no way actually contributes to radiogenic heat, primordial heat, or tidal dissipation. As a result, it is quite apparent that solar exergy is not embodied directly in any of these sources and it is inappropriate to characterize them as solar emergy or to characterize their equivalence ratio with solar exergy as a transformity (sej J^{-1}). These observations have lead to several clarifications in terminology (see Table 1). Since solar radiation is not directly 'embodied' in geothermal exergy, the ratio of geothermal exergy to solar radiation is not a transformity, but instead is a solar equivalence ratio (SER). Also, the result of multiplying a SER by the exergy of the geothermal source does not yield emergy, but instead, solar equivalent exergy. These are very important distinctions and are carried throughout this current work as well as the other papers in this special edition (Brown et al., 2016; Brown and Ulgiati, 2016a; Brown and Ulgiati, 2016b, Campbell, 2016; De Vilbiss et al., 2016).

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Table 1
Abbreviations used in this manuscript.

Abbreviation of symbol	Meaning
GEB	Geobiosphere emergy baseline
gej	gravitational emjoules
GPE	Gravitational potential energy
seJ	Solar equivalent joule
sej	Solar emjoule
SER	Solar equivalence ratio
$_{g}T_{S}$	Gravitational transformity of solar radiation
$_{g}T_{R}$	Gravitational transformity of radionuclide

As for Raugei's other assertion, while the heat released at Earth's surface is the result of the combined heat flows from primordial heat and radionuclide decay, these two sources of heat do not have the same properties and thus should not be expected to have the same SERs. Far from it, we agree with Raugei (2013), they should be expected to have different SERs. As a result of this supposition, we have undertaken this evaluation, using a novel approach to compute SERs for the four major radionuclides (^{238}U, ^{235}U, ^{232}Th, ^{40}K) responsible for a portion of the geothermal output of Earth.

1.1. Forward vs. backward computation

Our approach uses forward computation of the available energy required to synthesize the radionuclides. It is a departure from previous calculations of solar equivalence of the primary geobiosphere inputs as outlined by Odum (2000) and used by Brown and Ulgiati (2010, 2016a) as well as by Campbell (2016), which, in general, use backward calculation to establish an equivalence between the solar radiation and tidal dissipation and between solar radiation and geothermal flux. The difference between forward and backward computation is as follows. In a forward calculation one computes the available energy required to make something, in this case the energy required to synthesize the radionuclides. In backward calculation, one uses some form of equivalence between a given energy flux and another energy source, for instance, geothermal flux and solar radiation. While there are any number of ways this might be done, two common ways rely on either algebraic manipulation of equations (Odum, 2000; Brown and Ulgiati, 2010, 2016a) or through a third energy flux that can be related to both of the first two (Campbell, 2016).

In this study, we compute the gravitational exergy required to create the conditions for nuclear synthesis, a forward computation, and the gravitational exergy required to generate solar radiation, also a forward computation. Since both products (radionuclides and solar radiation) are the result of the dissipation of the same form of exergy (gravitational) a simple relationship can then be made between the ratios of gravitational exergy/radionuclide exergy and gravitational exergy/solar radiation, yielding an equivalence between solar radiation and radionuclide exergy, and generating a solar equivalence ratio for the latter.

1.2. Production of light and matter

The production of light and the synthesis of heavier forms of matter are simultaneous processes, and the resulting radiation and heavy elements are co-products. The energies involved in these reactions are well known. In our sun, production of solar radiation is primarily carried out by proton–proton chain reactions (fusion reactions) that combine hydrogen into helium. Nucleosynthesis of heavy radioactive elements does not occur in small stellar bodies like our sun, but instead primarily occurs in supernova stars many time more massive than the sun, catalyzed by gravitational collapse that releases the tremendous quantities of energy required to create the heaviest isotopes. In both cases, the driving force that creates the conditions for the production of light and radioisotopes is gravity. We assume equivalence between the gravitational exergy required to produce light in the sun and the gravitational exergy required for nucleosynthesis, albeit the latter requires much greater quantities.

2. Methods

2.1. Background

In this paper, we invoke some basic thermal and astrophysical concepts. Firstly, our emergy calculation relies heavily on the statistical mechanical concept of thermal energy, which uses a microscopic description of systems containing large number of particles. (Jacobs, 2013, p. 53) The various states of these particles give rise to whole system thermodynamic properties such as temperature, pressure, volume, and internal energy. Because of the assumption that particle numbers are large, we can treat the ensembles statistically and therefore use probability distributions to describe a system's state and derive its properties. (Jacobs, 2013, p. 53) The distribution we will use is the Boltzmann distribution as applied to an ideal gas (defined and explained in Jacobs (2013), pp. 62–70).

For the purposes of understanding stellar evolution, we use the statistical model of an ideal gas, which is based on the ideal assumptions of point-like (zero sized) particles with no intermolecular forces. (Blundell and Blundell, 2006, p. 8) This model relies on the concept of translational motion of particles, referring to each distinguishable particle's unique velocity given by its position and speed in all three spatial vectors. (Jacobs, 2013, p. 58) This leads naturally to the calculation of the translational kinetic energy of a particle as $1/2mv(x,y,z)^2$, and as we are concerned with the system as a whole, we will be using average translational kinetic energy of $1/2m\langle v^2\rangle$.

We know now that the universe contains many galaxies in which stars are constantly being born through gravitational collapse leading to the condensation of gas in the interstellar medium. (Law and Rennie, 2015a) This collapse produces the high temperatures at which stellar nucleosynthesis takes place and from which the heavier elements and starlight are together forged. This gravitational collapse occurs only when the absolute value of the gravitational potential energy (E) in a large area is greater than the internal energy of the gas (U) itself:

$$|E| > U.$$

This happens at the so called Jeans density, which for hydrogen is about 5E−17 kg m^{-3}, (Ryan and Norton, 2010, p. 177) and the collapse proceeds until hydrostatic equilibrium is reached whereby the increased pressure on the inner surface of the star balances that created by the force of gravity.

We use thermal energy as a proxy for gravity, the reasoning for which, comes from the understanding that as the gravitational force condenses the particles comprising a star, it is doing work (W) equal to the potential energy (E) input from Gravity (Panat, 2008, p. 6):

$$dW = dE$$

This is negative work done in the process of an approximate adiabatic compression of an ideal gas, so that (Panat, 2008, p. 8):

$$dU = -dW = -P * dV, \quad \text{and} \quad -PdV = NK_B dT,$$

and so we can see that the magnitude of the input of gravitational potential energy (E) is equal to the change in internal energy (U) of the star, and causes a proportionate increase in temperature (T):

$|dE| = dU \propto dT$ (we will use this relationship in methods, calculated as $U = 3K_BT/2$)

Modeling stellar gravitational collapse as an adiabatic compression of an ideal gas is a good assumption, and is the basis for the virial theorem, one of the cornerstones of stellar astrophysics. (Ryan and Norton, 2010, p. 179)

After gravity condenses hydrogen and helium to form a star, stellar evolution continues as the high temperatures of the plasma core give rise to the process of nucleosynthesis. (Law and Rennie, 2015b) At high temperatures, fusion reactions between atoms take place spontaneously, forming new and heavier elements as the nuclei of smaller atoms combine. In average stars such as our sun, the main fusion reactions we are concerned with involve the nucleosynthesis of helium from hydrogen, called "proton–proton chain" reactions. (Clayton, 1968) These are the primary reactions that produce the sunlight that fuels our biosphere here on Earth, and are of concern to us in this paper as we seek a gravitational transformity for sunlight.

Nuclear fusion is a stellar process that takes place during the life of a star, but even the largest stars do not produce elements heavier than Iron-56 during normal nucleosynthesis. The production of heavier isotopes occurs exclusively under the high energy conditions that accompany the end-of-life explosions of stellar cores, known as supernova. (Law and Rennie, 2015) These supernova are thought to occur once every thirty years in our galaxy, and can therefore account for the relative abundance of heavy isotopes. There are four of these isotopes in particular that are of special importance to emergy science, due to the significant radiogenic heating effect they have on planet Earth. In this paper, we will use thermal energy proxies for the gravitationally driven supernova to derive a gravitational transformity for these isotopes.

Because the fusion process requires gravitational exergy as a catalyzing input, the two co-products of solar radiation and radionuclide material can be expressed in terms of *gravitational emergy* (expressed as gravitational emjoules or *gej*),[1] which is the amount of gravitational exergy required in the steps of the fusion reactions. The result is two gravitational transformities, one for solar exergy and one for radiogenic exergy. The quantity of gravitational emergy required to produce a joule of exergy from radioactive decay is a gravitational transformity of radiogenic exergy (τ_R ; units = gej/J), while the gravitational emergy required to produce a joule of solar radiation is the gravitational transformity of solar radiation (τ_S ; units = gej/J). These two gravitational transformities can be used to compute a solar equivalence ratio (SER) between solar radiation and radiogenic exergy as follows:

$$\frac{\tau_R}{\tau_S} = SER$$

The units of which are:

$$\text{gej}/(\text{J}_{\text{Radiogenic}})/(\text{gej}/\text{J}_{\text{solar Rad.}}) = \text{J}_{\text{solar Rad.}}/(\text{J}_{\text{Radiogenic}})$$

Because the exergy of 1 atom of radionuclide is known (in the context of radiogenic heat, the energy released through radioactive decay), we can express a solar equivalence ratio of radiogenic exergy as solar equivalent joules per joule of heat released (seJ J^{-1}).

We assume equivalence between all gravitational potential, regardless of its conditions. This assumption allows us to equate the 'quality' of gravitational exergy as it applies to the sun, with the 'quality' of gravitational exergy as it applies to other stellar bodies. Further, we use thermal energy as a proxy for gravitational exergy required in nuclear fusion. Our rationale for this proxy is that gravitational exergy is translated into thermal energy by increasing the total pressure and temperature of the plasma gas inside stars. This thermal (molecular kinetic) energy is the operative energy at the site of fusion reactions, and results in collisions that enable nuclear processes to occur (the non-classical aspect of these nuclear processes is discussed in EndNote 1). We generalize that all internal (thermal) energy inside stars is instigated by gravitational exergy, through the mechanics previously described. Fortunately, approximate average temperatures at which these reactions occur, both in the sun and in supernova, are known and can be translated directly into estimates of thermal energies. Thermal energy is defined as the average translational kinetic energy possessed by free particles in a system. Classically, the kinetic energy of a particle is given as:

$$E_{ke} = \frac{1}{2}mv^2$$

Modeling stellar plasma as an ideal gas (Schwarzschild, 1958, p. 32), the average translational kinetic energy of a plasma is the expected value of the kinetic energy where the expected velocities of plasma particles follow a Maxwell–Boltzmann distribution (Jacobs, 2013, pp. 62–70) and,

$$\langle E_{ke} \rangle = \frac{1}{2}m\langle v^2 \rangle = \frac{1}{2}m\int_0^\infty v^2 \frac{4}{\sqrt{\pi}}\left(\frac{m}{2k_BT}\right)^{2/3} v^2 dv e^{(-mv^2)/(2k_BT)}$$
$$= \frac{3}{2}K_BT \quad (1)$$

where k_B is the Boltzmann constant equal to 1.38×10^{-23} J/K. A special case involves assessing the thermal energies associated with type II supernova explosions. The tremendous energies released in these events are catalyzed by gravitational collapse of a large end-of-life star's iron core (which must exceed the Chandrasekhar limit of 1.4 solar masses) into a very dense and hot neutron core (Ryan and Norton, 2010, p. 158). This collapse is very low entropy (Bethe et al., 1979), and after several milliseconds the core itself resembles a macroscopic nucleus and is effectively incompressible. The gravitational exergy of collapse is then rebounded into a shock wave that immediately precipitates r-process nucleosynthesis, which is responsible for creating all of the heaviest isotopes in the universe (Woosley et al., 1994; Witti et al., 1994; Thompson et al., 2001). Among these are U^{235}, U^{238}, and Th^{232} – three of four major radionuclides generating geothermal heat in the earth. The fourth, ^{40}K, is formed in explosive oxygen-burning process also ignited by type II supernova (Shimansky et al., 2003).

3. Results and discussion

3.1. Gravitational emergy of solar radiation

Solar radiation is the result of two fusion reactions (known as the p-p I and the p-p II chains) inside the sun. The p-p I chain (shown in Fig. 1) is a set of three reactions that fuse 4 protons into one helium atom, in the process releasing about 26.2 MeV of solar radiation, plus some neutrino energy (Clayton, 1968). The p-p II chain (shown

[1] According to the definition of emergy (cumulative exergy of one form required directly or indirectly to support a process), we introduce in this paper a new unit of emergy, gravitational emergy, defined as the cumulative gravitational exergy required to support the elementary conversion processes/products (i.e. solar radiation, matter, rotational kinetic energy, and heat). As a consequence, we also define a gravitational transformity for solar radiation and radiogenic exergies (τ_S and τ_R respectively as the gravitation emergy required per unit of available energy output (gej J^{-1}).

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Table 2
Solar radiation output at each stage of the p-p 1 cycle, gravitational potential exergy (GPE) required and gravitational emergy of solar radiation produced.

Reaction step	R[a]	Solar radiation output (MeV)[b]	GPE Required for reaction[c] (J)	Gravitational emergy (gej)
Reaction step 1				
$^1H + ^1H \rightarrow ^2H + e^+ + v_e$	2	2(1.02 – 0.25[d]) + 2(0.42) = 2.38	2(6.5E−16) = 13E−16	13E−16
$e^+ + e^- \rightarrow 2\gamma$	2			
Reaction step 2				
$^2H + ^1H \rightarrow ^3He + \gamma$	2	2(5.49) = 10.98	2(6.5E−16) = 13E−16	26E−16
Reaction step 3				
$^3He + ^3He \rightarrow ^4He + 2^1H$	1	12.86	6.5E−16	3.25E−15
Total		26.22		3.25E−15

[a] Column *R* is the number of times the reaction occurs.
[b] Radiation output taken from Clayton (1968; Table 5-1, p. 380).
[c] Gravitational potential exergy required is computed using Eq. (1) and sun's temperature of 15.7×10^6 K, yielding 3.25E−16 J/particle.
[d] 0.25 is the approximate neutrino (v_e) energy carried away during p-p I as estimated by Bahcall and Pinsonneault (2004). Neutinos do not contribute to sunlight, so they are not part of our radiation output and are subtracted here.

Table 3
Solar radiation output at each stage of the p-p II cycle, gravitational exergy required and gravitational emergy of solar radiation produced.

Reaction steps	R[a]	Solar radiation output[b] (MeV)	GPE Required for reaction[c] (J)	Gravitational emergy (gej)
Reaction 1				
$^1H + ^1H \rightarrow ^2H + e^+ + v_e$	3	3(1.02 – 0.25[d]) + 3(0.42) = 3.57	3(6.5E−16) = 19.5E−16	19.5E−16
$e^+ + e^- \rightarrow 2\gamma$	3			
Reaction 2				
$^2H + ^1H \rightarrow ^3He + \gamma$	3	3(5.49) = 16.47	3(6.5E−16) = 19.5E−16	3.9E−15
Reaction 3				
$^3He + ^3He \rightarrow ^4He + 2^1H$	1	12.86	6.5E−16	4.55E−15
Reaction 4				
$^3He + ^4He \rightarrow ^7Be + \gamma$	1	1.59	6.5E−16	5.2E−15
Reaction 5				
$^7Be + e^- \rightarrow ^7Li + v_e$	1	0(0.86)[e]	~	5.2E−15
Reaction 6				
$^7Li + ^1H \rightarrow ^4He + ^4He$	1	17.35	6.5E−16	5.85E−15
Total		51.85		5.85E−15

[a] Column *R* is the number of times the reaction occurs.
[b] Radiation output taken from Clayton (1968; Table 5-1, p. 380).
[c] Gravitational potential exergy required is computed using Eq. (1) and sun's temperature of 15.7×10^6 K, yielding 3.25E−16 J/particle.
[d] 0.25 is the approximate neutrino (v_e) energy carried away during p-p I as estimated by Bahcall and Pinsonneault (2004). Neutinos do not contribute to sunlight, so they are not part of our radiation output and are subtracted here.
[e] This is neutrino (v_e) energy only, therefore is not included.

in Fig. 2) is a set of six reactions that fuse four protons and one ^{4}He atom into two ^{4}He atoms, and involves the burning of beryllium and lithium-7, yielding 51.85 MeV. These two chain-reactions are detailed in the astrophysics classic: *Principles of Stellar Evolution and Nucleosynthesis* (Clayton, 1968). The reactions of the pp chains and the associated radiative outputs of each reaction are given in Tables 2 and 3.

To compute the gravitational emergy required to produce solar radiation we use the temperatures at which these two fusion reactions occur. Because the sun's core burns at 15.7×10^6 K (Ryan and

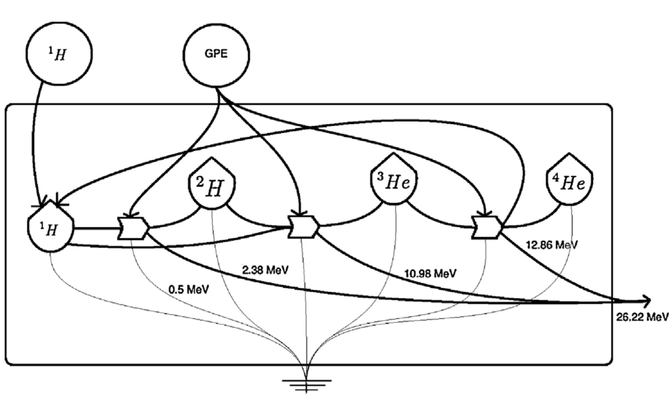

Fig. 1. Systems diagram of the p-p I chain of reactions showing the inputs of hydrogen and gravitational potential exergy (GPE) and the output of 26.22 MeV of solar exergy. Also note that a total of 0.5 MeV of exergy are carried away as a result of neutrino decay.

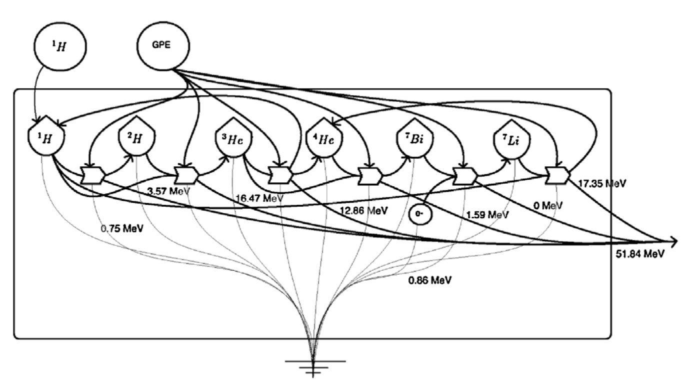

Fig. 2. Systems diagram of the p-p II chain of reactions showing the inputs of hydrogen and gravitational potential exergy (GPE) and the output of 51.84 MeV of solar exergy. Also note that a total of 1.61 MeV of exergy are carried away as a result of neutrino decay.

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Norton, 2010), the gravitational input for each reaction in the p-p I cycle can be computed using Eq. (1) as follows:

$$(3k_B(15.7\times10^6\,\mathrm{K})/2)*2 \quad \text{particles per nuclear reaction} = 6.5\times10^{-16}\,\mathrm{J}$$

The gravitational emergy in the final column of Tables 2 and 3 is the sum of the GPE input for each reaction.

The relative contributions that the p-p I and p-p II chain reactions make to solar radiation can be calculated based on their frequency of occurrence. The frequency of occurrence of the p-p I and the p-p II chains is 86% and 14% respectively (Bethe, 1939; Salpeter, 1952; Clayton, 1968):

Fraction of solar radiation from p-p I:

$$= 0.86(26.22)/[0.86(26.22) + 0.14(51.85)] = 0.76$$

Fraction of solar radiation from p-p II:

$$= 0.14(51.85)/[0.86(26.22) + 0.14(51.85)] = 0.24$$

3.2. Gravitational transformity of solar radiation

The total accumulated gravitational emergy over the final solar exergy output of each chain yields the gravitational transformity ($\mathrm{gej/J_{Solar}}$) for the p-p I and p-p II chains:p-p I (26.22 MeV total output):

$$3.25\times10^{-5}\,\mathrm{gej}/[26.22\,\mathrm{MeV}\times1.602\times10^{-13}\,\mathrm{J/MeV}]$$
$$= 7.74\times10^{-4}\,\mathrm{gej/J_{Solar}}$$

p-p II (51.85 MeV total output):

$$5.85\times10^{-5}\,\mathrm{gej}/[51.85\,\mathrm{MeV}\times1.602\times10^{-13}\,\mathrm{J/MeV}]$$
$$= 7.74\times10^{-4}\,\mathrm{gej/J_{Solar\ Rad.}}$$

These transformities imply that for every input of 0.000774 J (p-p I cycle) and 0.000704 J (p-p II cycle) of gravitational energy input to the stellar core, 1 J of solar radiation is produced. (*See EndNote 1 for further discussion*).

The p-p I chain, which generates about 76% of solar radiation, has a gravitational transformity of $7.74\times10^{-4}\,\mathrm{gej/J_{Solar}}$. The p-p II chain, which contributes about 24% of solar radiation, has a gravitational transformity of $7.04\times10^{-4}\,\mathrm{gej/J_{Solar}}$. A weighted average gravitational transformity for solar radiation is computed as follows:

$$gT_S = 0.76(7.74\times10^{-4}\,\mathrm{gej/J_{Solar}}) + 0.24(7.04\times10^{-4}\,\mathrm{gej/J_{Solar}})$$
$$= 7.57\times10^{-4}\,\mathrm{gej/J_{Solar}}$$

3.3. Gravitational emergy of radionuclides

Gravitational transformities for radionuclides were computed in much the same way as was done for solar radiation. Unlike the heavier-than-hydrogen elements and solar radiation generated inside our main-sequence sun, radionuclides are formed primarily through r-process nucleosynthesis in the very high temperatures generated by type II supernova. These temperatures were used to calculate the average energies of particles involved in the nuclear fusion reactions for each nuclide.

3.3.1. Potassium

Nucleosynthesis of ^{40}K, the third most significant contributor to radiogenic heat in the earth's crust and mantle, occurs in two steps (Fig. 3). The first step is a helium burning cycle that occurs post-hydrogen burning (p-p chains) in medium and large stellar cores, at temperatures of about 3×10^8 K (Meyer et al., 2008). This step first generates ^{12}C, and then ^{16}O, which is the dominant product of the helium burning cycle. ^{16}O is the substrate for the next phase of the ^{40}K production pathway, explosive oxygen burning, which occurs during type II supernova at temperatures of 3.5×10^9 K (Truran and Arnett, 1970). In these conditions oxygen is combined to sulfur, and sulfur recombines with free alpha particles to form calcium-40, which undergoes neutron capture and proton emission to become potassium-40 (Woosley et al., 1973). The overall set of reactions, are shown in Fig. 3 and summarized in Table 4.

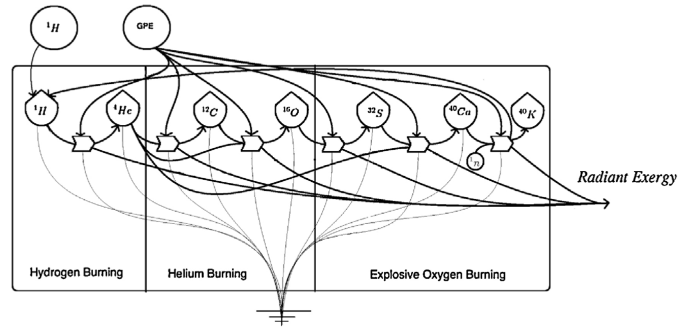

Fig. 3. Systems diagram of ^{40}K nucleosynthesis showing the input of gravitational potential exergy (GPE) and the input of hydrogen required during helium burning followed by explosive oxygen burning.

Both the helium burning phase and the explosive oxygen burning phase are important contributions to the gravitational emergy of ^{40}K. The gravitational emergy required for these two phases (using Eq. (1)) correspond to $(3/2)k_B(3E8\,\mathrm{K}) = 6.21\times10^{-15}$ gej, and $(3/2)k_B(3.5\times10^9\,\mathrm{K}) = 7.25\times10^{-14}$ gej. The first reactant however, is ^{4}He, for which we have two values for emergy/atom (one from the p-p I chain, another from p-p II). An average of these based on frequency of occurance is used to assign emergy to ^{4}He in the first reaction step: $0.86(3.25\times10^{-15}) + 0.14(5.85\times10^{-15}) = 3.6\times10^{-15}$. The third column in Table 4 lists the gravitational emergy required for each reaction depending on whether it is helium burning (6.21×10^{-15}) or explosive oxygen burning (7.25×10^{-14}). The gravitational emergy in the final column is the sum of the GPE required and the gravitational emergy of the reactants.

The exergy of ^{40}K, released by its radioactive decay, is 0.7 MeV (Stacey and Davis, 2008). Using the total gravitational emergy (6.93×10^{-13} gej) from Table 4, the gravitational transformity for the radioactive decay of ^{40}K is:

$$(6.93\times10^{-13}\,\mathrm{gej})(0.7\,\mathrm{MeV}*1.6\times10^{-13}\,\mathrm{J/MeV}) = 6.19\,\mathrm{gej/J_{40\,K}}$$

3.3.2. Heavy radionuclides (^{232}Th, ^{235}U, and ^{238}U)

The rare, heavy radionuclides (^{232}Th, ^{235}U, and ^{238}U) are the result of r-process nucleosynthesis, a process of rapid neutron capture by heavy "seed-nuclei". To compute the gravitational emergy required to synthesize the heavy radionuclides, not only are the thermal energies of r-process nucleosynthesis, necessary (calculated as 1.22×10^{-13} J, see *EndNote 2*), but also the emergy of the other required inputs (alpha particles and the seed nuclei) must be included. The heaviest and most likely seed nuclei required for uranium and thorium formation is peak iron, which is the final product of non-explosive nucleosynthesis and signifies the end of stellar fuel. (Pagel, 2009) The ^{56}Fe remaining after disentegration of the core in the collapse phase can form the seed nuclei for subsequent neutron and alpha particle capture in the hot bubble of high density neutrino flux, during the initial moments of supernova.

Gravitational emergy required for alpha particles (helium nuclei) was computed above (3.6×10^{-15} gej/atom). What remains is to compute the gravitational emergy required to synthesize ^{56}Fe.

Table 4
Reaction steps in the nucleosynthesis of ^{40}K showing required gravitational potential exergy input and the gravitational emergy of the products.

Reaction step	*R*[a]	GPE required per reaction step (J)	Gravitational emergy of reactants (gej)	Gravitational emergy (gej)
Initial input of 3(^{4}He)	2		2*3*(3.6E−15) = 2.16E−14	2.16E−14
^{4}He + ^{4}He → ^{8}Be	2	2[2(6.21E−15)] = 2.48E−14		4.64E−14
^{4}He + ^{8}Be → ^{12}C + y	2	2[2(6.21E−15)] = 2.48E−14		7.13E−14[b]
Input of 2[^{12}C] & 2[^{4}He]	1		7.13E−14 + 2(3.6E−15)	7.85E−14
^{12}C + ^{4}He → ^{16}O + y	2	2[2(6.21E−15)] = 2.484E−14		1.03E−13[c]
Input of 2(^{16}O)	1		1.03E−13	1.03E−13
^{16}O + ^{16}O → ^{32}S	1	2(7.25E−14) = 1.45E−13		2.48E−13
Input of ^{32}S and 2(^{4}He)	1		2.48E−13 + 2(3.6E−15)	2.55E−13
^{32}S + ^{4}He → ^{36}Ar^{36}Ar + ^{4}He → ^{40}Ca	1	2(7.25E−14) = 1.45E−13		4.00E−13
	1	2(7.25E−14) = 1.45E−13		5.45E−13
Input of ^{40}Ca	1		5.45E−13	5.45E−13
^{40}Ca + n → ^{40}K + p	1	2(7.25E−14) = 1.45E−13	1.45E−13	6.9E−13
Total gravitational emergy of ^{40}K				**6.9E−13**

[a] Column "*R*" is the number of times the reaction occurs.
[b] Gravitational emergy required to produce two ^{12}C atoms.
[c] Gravitational emergy required to produce two ^{16}O atoms.

Table 5
Reaction steps in the non-explosive nucleosynthesis of ^{56}Fe showing required GPE input, Gravitational emergy of the reactants, and the cumulative emergy of the products.

Reaction step	*R*[a]	GPE required per reaction[b] (J)	Gravitational emergy of reactants (gej)	Gravitational emergy (gej)
Helium burning				
Initial input of 3(^{4}He)	2		2*3*(3.6E−15) = 2.16E−14	2.16E−14
^{4}He + ^{4}He → ^{8}Be	2	2[2(6.21E−15)] = 2.48E−14		4.64E−14
^{4}He + ^{8}Be → ^{12}C + y	2	2[2(6.21E−15)] = 2.48E−14		7.12E−14[c]
Carbon burning				
Input of 2[^{12}C] and 2[^{4}He]	1		7.12E−14 + 2(3.6E−15)	7.84E−14
^{12}C + ^{4}He → ^{16}O + y	2	2[2(6.21E−15)] = 2.484E−14		1.03E−13[d]
Oxygen burning				
Input of 2(^{16}O)	1		1.03E−13	1.03E−13
^{16}O + ^{16}O → ^{28}Si + α	1	2(3.11E−14) = 6.22E−14		1.65E−13
Silicon burning				
Input of ^{28}Si and 7(^{4}He)	1		1.65E−13 + 7(3.6E−15)	1.91E−13
^{28}Si + 7(^{4}He) → ^{56}Fe	1	7[2(5.59E−14)] = 7.83E−13[e]		9.74E−13
Total gravitational emergy of ^{56}Fe				**9.74E−13**

[a] Column "*R*" is the number of times the reaction occurs.
[b] The temperatures at which these hydrostatic burning stages operate and their required gravitational exergy are from Woosley et al. (2002).
[c] Gravitational emergy required to produce two ^{12}C.
[d] Gravitational emergy required to produce two ^{16}O.
[e] Each alpha particle is considered to be associated with an individual reaction, thus we have 7 reactions of 2 particles per reaction.

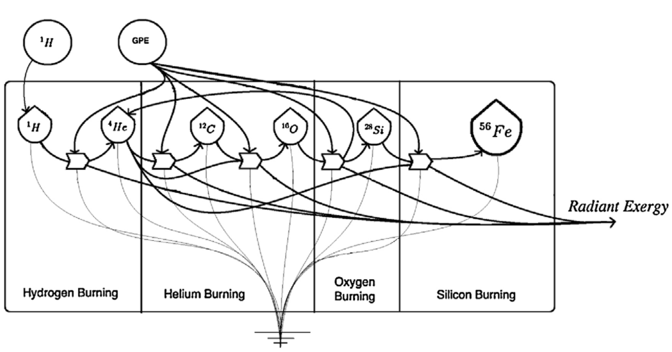

Fig. 4. Systems diagram of the non-explosive nucleosynthesis of ^{56}Fe showing the input of gravitational potential exergy (GPE) and the input of hydrogen for the first reaction that produces helium, followed by the required reactions during helium, oxygen and silicon burning.

The emergy of seed-nuclei ^{56}Fe is computed based on the exergies required for its formation, a cascade of reactions occuring at various temperatures, that are the normal hydrostatic burning stages in presupernova stars (Fig. 4).

Table 5 lists the reaction steps and the gravitational emergy reguired to synthesize ^{56}Fe. As shown in Fig. 4, prior to helium burning, there is a necessary input of helium, which is shown as the initial input of 3 ^4He atoms with a gravitational emergy of 3.6×10^{-15} gej each. The helium burning reaction occurs twice. The required GPE per reaction in this step is (2 reactions)*(2 atoms)*(6.21×10^{-15} J/atom). The output of the helium burning step is two ^{12}C atoms which are assigned the gravitational emergy of 7.12×10^{-14} gej. In the carbon burning stage, the two ^{12}C are combined with two ^{4}He and the reaction occures twice, yielding two atoms of ^{16}O, which are assigned the gravitational emergy of 1.03×10^{-15} gej. The subsequent oxygen-burning stage yields ^{28}Si which is assigned 1.65×10^{-13} gej. When the oxygen fuel is exhausted, the final stage of Si-burning begins, which is actualy just a succession of alpha-capture reactions, yielding ^{56}Fe, which is assigned the total emergy (9.74×10^{-13} gej).

With the computation of the gravitational emergy required for ^{56}Fe synthesis, gravitational transformities (gT_R)[2] for the r-process radionuclides can be computed. The ^{232}Th reaction is as follows:

$$^{56}\text{Fe} + 32\alpha + 48\text{n} \rightarrow {}^{232}\text{Th},$$

[2] We define gravitational transformity as the gravitational exergy required directly and indirectly supporting a process of production, abbreviated (*gT*) to differentiate it from solar transformities (*T*).

Table 6
Solar equivalence ratios (SER) for radionuclides.

Isotope	Gravitational transformity (gej/J)	SER[a] (seJ/J)
^{40}K	6.19	8.2E+3
^{232}Th	3.18	4.2E+3
^{235}U	2.93	3.9E+3
^{238}U	2.80	3.7E+3

[a] Computed by dividing isotope gravitational transformity by the gravitational transformity of solar exergy (7.57E−4 gej/J).

and the emergy/atom of thorium is:

$$9.74\times10^{-13}\,\text{gej}+32(3.6\times10^{-15}\,\text{gej})+(32+48)[2(1.22\times10^{-13}\,\text{gej})]=2.06\times10^{-11}\,\text{gej}.$$

The energy released during complete decay of ^{232}Th ($^{232}\text{Th}\rightarrow6\alpha+4\beta+^{208}\text{Pb}+\nu_e$) is 42 MeV, with 1.5 MeV neutrino energy (Stacey and Davis, 2008). The neutrino energy carried off must be subtracted, so we have a gravitational transformity for ^{232}Th as follows:

$$gT_{232\text{Th}}=(2.06\times10^{-11}\,\text{gej})/(40.5\,\text{MeV})=3.18\,\text{gej/J}_{232\text{Th}}.$$

The ^{235}U reaction,

$$^{56}\text{Fe}+33\alpha+47\text{n}\rightarrow{}^{235}\text{U},$$

yields an emergy/atom of ^{235}U equal to:

$$9.74\times10^{-13}\,\text{gej}+33(3.6\times10^{-15}\,\text{gej})+(33+47)[2(1.22\times10^{-13}\,\text{gej})]=2.06\times10^{-11}\,\text{gej}.$$

The energy released during complete decay of ^{235}U ($^{235}\text{U}\rightarrow7\alpha+4\beta+^{207}\text{Pb}+\nu_e$) is 46 MeV, with 2.1 MeV neutrino energy (Stacey and Davis, 2008). The neutrino energy carried off must be subtracted, so we have a gravitational transformity for ^{235}U as follows:

$$gT_{235\text{U}}=(2.06\times10^{-11}\,\text{gej})/(43.9\,\text{MeV})=2.93\,\text{gej/J}_{235\text{U}}.$$

The ^{238}U reaction,

$$^{56}\text{Fe}+33\alpha+50\text{n}\rightarrow{}^{238}\text{U},$$

yields an emergy/atom of ^{238}U equal to:

$$9.73\times10^{-13}\,\text{gej}+33(3.6\times10^{-15}\,\text{gej})+(33+50)[2(1.22\times10^{-13}\,\text{gej})]=2.13\times10^{-11}\,\text{gej}.$$

The energy released during complete decay of ^{238}U ($^{235}\text{U}\rightarrow8\alpha+6\beta+^{206}\text{Pb}+\nu_e$) is 52 MeV, with 4.3 MeV neutrino energy (Stacey and Davis, 2008). The neutrino energy carried off must be subtracted, so we have a gravitational transformity for ^{238}U as follows:

$$gT_{238\text{U}}=(2.13\times10^{-11}\,\text{gej})/(47.7\,\text{MeV})=2.80\,\text{gej/J}_{238\text{U}}.$$

3.4. Solar equivalence ratios of radionuclides

A solar equivalence ratio for radionuclides (SER: units = seJ/J) can be computed using the gravitational transformity for solar radiation, and gravitational transformities for radionuclides and assuming an equivalence between the two ratios formed by the gravitational transformities as follows:

$$\text{SER}=\frac{\text{gej/J}_{\text{rodionuclide}}}{\text{gej/I}_{\text{solar}}}=\frac{\text{sej}}{\text{J}_{\text{rodionuclide}}}$$

For ^{40}K, the SER is $(6.19\,\text{gej/J}_{40\text{K}})/(7.57\times10^{-4}\,\text{gej/J}_{\text{Solar}})$ = 8177 seJ/J.

For ^{232}Th, the SER is $(3.18\,\text{gej/J}_{232\text{Th}}/\text{J})/(7.57\times10^{-4}\,\text{gej/J}_{\text{Solar}})$ = 4200 seJ/J.

For ^{235}U, the SER is $(2.93\,\text{gej/J}_{235\text{U}}/\text{J})/(7.57\times10^{-4}\,\text{gej/J}_{\text{Solar}})$ = 3870 seJ/J.

For ^{238}U, the SER is $(2.80\,\text{gej/J}_{238\text{U}})/(7.57\times10^{-4}\,\text{gej/J}_{\text{Solar}})$ = 3698 seJ/J

Table 6 summarizes the solar equivalence ratios for the radionuclides.

4. Conclusions

As is well known, solar radiation and radionuclides are generated by the convergence of gravitational potential exergy. By using gravitational exergy required to produce both solar radiation and radionuclides, we have introduced a modification of the procedures to compute emergy and transformity, remaining within the definitions of both. We have defined the cumulative gravitational exergy required to support the elementary conversion processes/products (i.e. solar radiation, matter, rotational kinetic energy, and heat) as gravitational emergy. As a consequence, we also define a gravitational transformity for solar radiation and radiogenic exergies (gT_S and gT_R) respectively. We have then introduced a method of indirectly computing the solar equivalence ratios of the radiogenic isotopes using the gravitational exergy required to produce both solar radiation and radiogenic heat. These results can be seen as a major contribution to a recalculation of the geobiosphere emergy baseline by referencing the primary biosphere flows (solar radiation, radiogenic heat, primordial heat and Earth's rotational KE) to the gravitational potential exergy necessary to produce them.

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EndNote 1.

This may seem odd at first, until we recognize two important factors that cause this UEV to be smaller than we would intuitively expect. The first is that nuclear fusion is a quantum mechanical process that involves a phenomenon called quantum tunneling, whereby a proton can sometimes tunnel over the coulomb barrier in another atom even if it does not have sufficient energy to overcome the coulomb repulsion force according to classical electrodynamics. In fact, we can calculate the energy per reaction required to overcome the coulomb barrier for proton–proton fusion as the Coulomb potential:

$$U=\frac{e^2}{4\pi r_{\epsilon 0}}=\frac{(9\times((10^9\,\text{Nm}^2)/c^2))(1.60\times10^{-19}\,\text{C})^2}{3\times10^{-15}\,\text{m}}=7.68\times10^{-14}\,\text{J}$$

This is two orders of magnitude greater than our average energies of 6.5×10^{-16} J and in fact corresponds to temperatures of $(2/3)(7.68\times10^{-14})/(2k_B)=1.9\times10^9$ K, which the sun does not reach. The reason that fusion reactions occur in significant quantities in the sun is that the core contains so many protons that the probability of fusion occurring is very high.

The second reason for such a low UEV is that, although very little mass is lost in the fusion reactions, and the total number of protons is always conserved, the small amount of mass that is transmuted during each reaction has relatively large energies according to the energy–mass equivalence $E = mc^2$.

EndNote 2.

As was shown by the ground-breaking work of Margaret and Geoffrey Burbidge, William Fowler, Fred Hoyle, and Alistair Cameron, the heavy radiogeneics (^{238}U, ^{235}U, and ^{232}Th) are all products of r-process nucleosynthesis, whereby seed nuclei under extreme conditions of high temperatures and high neutron flux rapidly capture neutrons to form the heaviest elements. (Burbidge et al., 1957; Cameron, 1957). Core-collapse supernova are widely considered to be the main progenator of r-process nucleosynthesis, but the exact site of the supernova at which this takes place is debated (Matthews and Cowan, 1990; Cowan et al., 1991; Takahashi et al., 1994). Currently, the primary opinion is that the ideal conditions are most likely met in a “hot bubble” region of high density neutrino flux at the surface of a newly formed protoneutron star (Meyer et al., 1992; Woosley et al., 1994; Surman et al., 2013).

All stars greater than 8 solar masses undergo gravitational collapse of the eventual iron core, resulting in type II supernova explosions (Weaver and Woosley, 1993). In these core-collapse supernova, the iron peak nuclei disintegrate into alpha particles, then protons, and neutrons. The collapsing electron pressure from the previously degenerate state causes an increasing neutronization of matter, which results in the production of neutrinos ($p^+ + e^- \rightarrow n + \nu_e$) and a neutron core that itself becomes degenerate. The protoneutron star at nuclear density rebounds the energy of gravitational collapse. The shock wave perpetuated outward has energies on the order of 2×10^{46} J (2×10^{53} erg) (Thielemann et al., 1998). However, 99% of this energy is carried by neutrinos, leaving only 1% as shock-wave-generated thermal energy. This thermal energy takes the form of a rapidly expanding, high-energy, high-entropy bubble, which is likely the site of r-process synthesis of heavy nucleides. Temperatures here are related to the rebounded shock energy ($E_0 = 0.01(2 \times 10^{46}\,\text{J}) = 2 \times 10^{44}\,\text{J}$) by the radiation energy density equation:

$$E_0 = VaT^4$$

where V is the volume ($4\pi r^3/3$), and a is the radiation density constant $7.56 \times 10^{-16}\,\text{J}\,\text{m}^{-3}\,\text{K}^{-4}$. For a radius of 3700 km – corresponding to r-process radius of a 20 solar mass star (Pagel, 2009) – this shock energy translates to average r-process temperatures of 5.9×10^9 K, and energies of 1.22×10^{-13} J.

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